

(h, k) -Dichotomy on time scales and its application to Volterra integro-dynamic systems

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ABSTRACT. In this paper, we focus on the concept of (h, k) -dichotomies on time scales and obtain some results regarding (h, k) -dichotomies which enable us to study periodic solutions of dynamic systems on time scales. In the setup of the main results, we propose an integrability condition for (h, k) -dichotomies and show that the Green function is unique up to period T . As an implementation of the theoretical findings, we analyze a certain kind of delayed Volterra integro-dynamic system on unbounded time scales and propose sufficient conditions for the existence of periodic solutions due to the alternative periodicity concept on time scales based on shift operators and fixed point theory.

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