

Differential-Boundary Equations with Algebraic Terms and Associated Boundary Value Problems

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ABSTRACT. In this paper, differential-boundary operators with algebraic terms are considered. The question is posed: how many and what kind of conditions should be added to the original equation for the resulting problem to have a unique solution in some space? What is the structure of the inverse operator? Comprehensive answers to these questions are provided. Illustrative examples are given.

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