CARPATHIAN J. MATH.
Volume **41** (2025), No. 4,
Pages 981-998

Online version at https://www.carpathian.cunbm.utcluj.ro/
ISSN 1584-2851 (Print edition); ISSN 1843-4401 (Electronic)
DOI: https://doi.org/10.37193/CJM.2025.04.09

Multiperiodic solutions of linear systems with a differentiation operator in the directions of the diagonal of the space of independent variables

BIBIGUL OMAROVA[1], ZHAISHYLYK SARTABANOV[2], ZHUMAGAZIYEV AMIRE[1,4], AND GULSEZIM AITENOVA[3]

ABSTRACT. In the paper, we generalize the methods of studying periodic solutions of linear systems of ordinary differential equations to linear multiperiodic systems with a differentiation operator along the main diagonal of the space of independent variables based on the study of the characteristic systems of partial differential operator on a multidimensional cylindrical manifold. A point on this manifold with time coordinates moves along the helix of a circular cylindrical surface , which is a periodic characteristic of the diagonal differentiation operator. In conclusion, we established the conditions for the multiperiodicity of solutions of linear systems with such an operator; justified the criteria for the multiperiodicity of solutions of systems associated with the matrix of those systems; determined the connections between periodic solutions of homogeneous and inhomogeneous systems based on the concept of conjugate systems; and provided integral representations of multiperiodic solutions using the Green function.

ACKNOWLEDGMENTS

This research was funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan grant number AP19676629

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Received: 13.12.2024. In revised form: 25.01.2025. Accepted: 26.05.2025
2020 *Mathematics Subject Classification.* 35B10, 26B10, 35F10, 35R20, 34C25, 34B27.
Key words and phrases. *cylindrical surface, helical line, periodic characteristic, multiperiodicity, matricant, conjugate systems, differentiation operator, orthogonality, Green's function.*
Corresponding author: Zhumagaziyev Amire; azhumagaziyev@zhubanov.edu.kz

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[1]Heriot-Watt International Faculty
K. Zhubanov Aktobe Regional University
030000, Aktobe, Kazakhstan
Email address: bibigul.zharbolkyzy@gmail.com, azhumagaziyev@zhubanov.edu.kz

[2]Department of Mathematics
K. Zhubanov Aktobe Regional University Aktobe
030000, Aktobe, Kazakhstan
Email address: sartabanov42@mail.ru

[3]Educational program for the training of teachers of mathematics and physics
M. Utemisov West Kazakhstan University
090000, Uralsk, Kazakhstan
Email address: gulsezim-88@mail.ru

[4]Department of mathematics and methods of teaching mathematics
Kh.Dosmukhamedov Atyrau University
060000, Atyrau, Kazakhstan
Email address: azhumagaziyev@zhubanov.edu.kz