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The Stinespring-Wittstock theory for variable exponential completely bounded maps

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ABSTRACT. We establish the fundamental factorization theorem for operators between Banach spaces under the condition of the variable exponent $p(\cdot)$-completely boundedness. Assuming $X,\quad Y$ and $\tilde{X},\quad \tilde{Y}$ be separable Banach spaces, we show that the variable exponent $p(\cdot)$-completely bounded mapping $\Lambda\ :\quad S \to LB\left(\tilde{X},\ \tilde{Y}\right), S \subset LB(X,\ Y)$ can be presented in the form $\Lambda(a) = \theta_2 \hat{\pi}(a)\theta_1$, where $\theta_1\ :\quad \tilde{X} \to V/N$, $\theta_2\ :\quad \breve{V}/\breve{N} \to \tilde{Y}$, and $\hat{\pi}(a)\ :\quad V/N \to \breve{V}/\breve{N}$ such that $\|\theta_1\|\,\|\theta_2\| \leq \|\Lambda\|_{p(\cdot)-cb}$, the reverse is also true.

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