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Enhancing the applicability of a family of fifth-order iterative methods

AJIL KUNNARATH[1], SANTHOSH GEORGE[1], P. JIDESH [1], AND IOANNIS K. ARGYROS[2]

ABSTRACT. We develop a class of fifth order methods introduced by Ali Zein (2023). There are many efficient fifth order methods that are special cases of this class. We present the method in a more abstract setting of a Banach space. The semilocal convergence is discussed first, and using the semilocal analysis, we obtain a ball containing the solution. The local convergence analysis does not depend on the Taylor series, and we relax the differentiability conditions on the function involved. Our assumptions for obtaining the convergence order are independent of the solution; earlier studies use assumptions involving the solution for local convergence analysis. We considered several numerical examples in chemical and physical sciences to analyze the behavior of the method. The dynamics of the method are studied.

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Corresponding author: Ajil Kunnarath;ajilk.227ma500@nitk.edu.in

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Email address: ajilk.227ma500@nitk.edu.in

Email address: sgeorge@nitk.edu.in

Email address: jidesh@nitk.edu.in

Email address: iargyros@cameron.edu

[1] Department of Mathematical and Computational Sciences, National Institute of Technology Karnataka, Surathkal, Mangalore, Karnataka, India, PIN-575 025

[2]Department of Computing and Mathematical Sciences, Cameron University, Lawton, OK 73505, USA