

Gallai's property for Archimedean tiling graphs on the torus

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ABSTRACT. This paper investigates Gallai's property for Archimedean tiling graphs on the torus. Moreover, we find a C_2^2 -graph in $(3^4.6)$ of a smaller order than the one presented in [Nadeem, M. F.; Iqbal, H.; Siddiqui, H. M. A.; Azeem, M. Intersecting longest cycles in Archimedean tilings. *Algorithmica*, **85** (2023), 2348-2362].

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