CARPATHIAN J. MATH.
Volume **42** (2026), No. 1,
Pages 153-170

Online version at https://www.carpathian.cunbm.utcluj.ro/
Print Edition: ISSN 1584 - 2851; Online Edition: ISSN 1843 - 4401
DOI: https://doi.org/10.37193/CJM.2026.01.11

Finite-Time H_∞ Control for Conformable Fractional-Order Nonlinear Systems with Time Delays

NGUYEN THI HUE[1], MAI VIET THUAN[2], NGUYEN HUU SAU[3], AND NGUYEN THI THANH HUYEN[4]

ABSTRACT. This paper studies finite-time H_∞ control for conformable fractional-order nonlinear systems with time delays. Unlike Caputo derivatives, conformable derivatives lack memory effects, affecting stability. Existing work on H_∞ control mainly covers integer-order and Caputo fractional systems, with little focus on conformable systems with delays. Using Lyapunov methods and the conformable Laplace transform, we derive linear matrix inequalities (LMIs) that ensure finite-time stability and robust H_∞ performance. A state-feedback control law is designed to improve system robustness. Numerical simulations validate the approach against disturbances and delays.

ACKNOWLEDGMENTS

The authors would like to express their sincere gratitude to the anonymous reviewers and editors for their thorough review and valuable recommendations, which have significantly contributed to improving the manuscript. The research is funded by the Ministry of Education and Training of Vietnam under grant number B2025-CTT-06.

REFERENCES

[1] Amato, F.; Ambrosino, R.; Ariola, M.; Cosentino, C.; De Tommasi, G. Finite-time stability and control. Lecture Notes in Control and Information Sciences, 453. *Springer, London*, 2014.
[2] Abreu-Blaya, R.; Fleitas, Alberto; Nápoles Valdés, J. E.; Reyes, R.; Rodríguez, J. M.; Sigarreta, J. M. On the conformable fractional logistic models. On the conformable fractional logistic models. *Math. Methods Appl. Sci.* **43** (2020), no. 7, 4156–4167.
[3] Abdeljawad, T. On conformable fractional calculus. *J. Comput. Appl. Math.* **279** (2015), 57–66.
[4] Balcı, E.; Öztürk, İ.; Kartal, S. Dynamical behaviour of fractional order tumor model with Caputo and conformable fractional derivative. *Chaos Solitons Fractals* **123** (2019), 43–51.
[5] Boyd, S.; El Ghaoui, L. ; Feron, E.; Balakrishnan, V. Linear Matrix Inequalities in System and Control Theory. *SIAM Studies in Applied Mathematics, 15. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA*, 1994.
[6] Binh, T. N.; Sau, N. H.; Huyen, N. T. T.; Thuan, M. V. Guaranteed cost control of delayed conformable fractional-order systems with nonlinear perturbations using an event-triggered mechanism approach. *Internat. J. Systems Sci.* **56** (2025), no. 12, 2991–3008.
[7] Chung, W. S. Fractional Newton mechanics with conformable fractional derivative. *J. Comput. Appl. Math.* **290** (2015), 150–158.
[8] Gahinet, P.; Nemirovskii, A.; Laub, A. J.; Chilali, M. LMI control toolbox for use with MATLAB. Natick, MA: The MathWorks, Inc., 1995.
[9] He, S.; Liu, P. Finite-time H_∞ fuzzy control of nonlinear jump systems with time delays via dynamic observer-based state feedback. *IEEE Trans. Fuzzy Syst.* **20** (2011), no. 4, 605–614.
[10] Huyen, N. T. T.; Thanh, N.T.; Sau, N. H.; Binh, T. N.; Thuan, M. V. Mixed H_∞ and passivity performance for delayed conformable fractional-order neural networks. *Circuits Syst. Signal Process.* **42** (2023), no. 9, 5142–5160.

Received: 27.05.2025. In revised form: 30.09.2025. Accepted: 14.10.2025
2020 *Mathematics Subject Classification.* code, code.
Key words and phrases. *Finite-time H_∞ control, Conformable Laplace transform, Time delays, Fractional conformable exponential function .*
Corresponding author: Mai Viet Thuan; thuanmv@tnus.edu.vn

[11] Jin, Q.; Sun, L.; Yang, R. Finite-time H_∞ control for a class of Takagi-Sugeno fuzzy singular systems. *Proc. Inst. Mech. Eng. I J. Syst. Control Eng.* **238** (2024), no. 4, 737–743.
[12] Khalil, R.; Al Horani, M.; Yousef, A.; Sababheh, M. A new definition of fractional derivative. *J. Comput. Appl. Math.* **264** (2014), 65–70.
[13] Li, Y.; Liu, L.; Feng, G. Finite-time $\mathcal{H}_\infty$ controller synthesis of T-S fuzzy systems. *IEEE Trans. Syst. Man Cybern. Syst.* **50** (2018), no. 5, 1956–1963.
[14] Luo, L.; Li, L.; Cao, J.; Abdel-Aty, M. Fractional exponential stability of nonlinear conformable fractional-order delayed systems with delayed impulses and its applications. *J. Franklin Inst.* **362** (2025), no. 1, Paper No. 107353, 23 pp.
[15] Ma, X.; Wu, W.; Zeng, B.; Wang, Y.; Wu, X. The conformable fractional grey system model. *ISA Trans.* **96** (2020), 255–271.
[16] Mai, V. T.; Nguyen, T. H. T.; Nguyen, H. S.; Nguyen, T. T. H. New results on H_∞ control for nonlinear conformable fractional order systems. *J. Syst. Sci. Complex.* **34** (2021), no. 1, 140–156.
[17] Naifar, O.; Jmal, A.; Ben Makhlouf, A. Non-fragile H_∞ observer for Lipschitz conformable fractional-order systems. *Asian J. Control* **24** (2022), no. 5, 2202–2212.
[18] Niamsup, P.; Thanh, N. T.; Phat, V. N. Finite-time H_∞ control of linear singular fractional differential equations with time-varying delay. *IMA J. Math. Control Inform.* **39** (2022), no. 2, 773–788.
[19] Peng, X.; Wu, H. Non-fragile robust finite-time stabilization and H_∞ performance analysis for fractional-order delayed neural networks with discontinuous activations under the asynchronous switching. *Neural Comput. Appl.* **32** (2020), no. 8, 4045–4071.
[20] Sadek, L.; Abouzaid, B.; Sadek, E. M.; Alaoui, H. T. Controllability, observability and fractional linear-quadratic problem for fractional linear systems with conformable fractional derivatives and some applications. *Int. J. Dyn. Control* **11** (2023), no. 1, 214–228.
[21] Shen, H.; Li, F.; Yan, H.; Karimi, H. R.; Lam, H. K. Finite-time event-triggered $\mathcal{H}_\infty$ control for T-S fuzzy Markov jump systems. *IEEE Trans. Fuzzy Syst.* **26** (2018), no. 5, 3122–3135.
[22] Souahi, A.; Makhlouf, A. B.; Hammami, M. A. Stability analysis of conformable fractional-order nonlinear systems. *Indag. Math. (N.S.)* **28** (2017), no. 6, 1265–1274.
[23] Thanh, N. T.; Phat, V. N. Improved approach for finite-time stability of nonlinear fractional-order systems with interval time-varying delay. *IEEE Trans. Circuits Syst. II Express Briefs* **66** (2018), no. 8, 1356–1360.
[24] Tarasov, V. E. "Conformable fractional" derivatives and integrals are integer-order operators: Physical and geometrical interpretations, applications to fractal physics. *Chaos Solitons Fractals* **192** (2025), Paper No. 116066, 9 pp.
[25] Thuan, M. V.; Sau, N. H.; Huyen, N. T. T. Finite-time H_∞ control of uncertain fractional-order neural networks. *Comput. Appl. Math.* **39** (2020), no. 2, Paper No. 59, 19 pp.
[26] Trang, T. T.; Phat, V. N.; Adly, S. Finite-time stabilization and H_∞ control of nonlinear delay systems via output feedbacks. *J. Ind. Manag. Optim.* **12** (2016), no. 1, 303–315.
[27] Xiao, S.; Li, J. Exponential stability of impulsive conformable fractional-order nonlinear differential system with time-varying delay and its applications. *Neurocomputing* **560** (2023), Paper No. 126845, 8 pp.
[28] Xu, L.; Bao, B.; Hu, H. Stability of impulsive delayed switched systems with conformable fractional-order derivatives. *Internat. J. Systems Sci.* **56** (2025), no. 6, 1271–1288.
[29] Xu, J.; Cui, Y.; Rui, W. Innate character of conformable fractional derivative and its effects on solutions of differential equations. *Math. Methods Appl. Sci.* **48** (2025), no. 9, 9414–9429.
[30] Younis, J.; Ahmed, B.; AlJazzazi, M.; Al Hejaj, R.; Aydi, H. Existence and uniqueness study of the conformable Laplace transform. *J. Math.* **2022**, Art. ID 4554065, 7 pp.

[1] Faculty of Fundamental and Applied Sciences, Thai Nguyen University of Technology
Email address: nguyenthihue@tnut.edu.vn

[2] Department of Mathematics and Informatics, TNU–University of Sciences
Email address: thuanmv@tnus.edu.vn

[3] Faculty of Fundamental Sciences, Hanoi University of Industry, 298 Caudien, Tay Tuu Ward, Hanoi, Vietnam
Email address: nguyensau@haui.edu.vn

[4] Department of Mathematics and Informatics, TNU–University of Sciences
Email address: huyen.ntt@tnus.edu.vn