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# Distinguishing Number of Inverse Graphs of Finite Groups: Labeling the Symmetry

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ABSTRACT. The distinguishing number of a graph measures its symmetry, and it is defined as the smallest number of labels needed to uniquely identify its vertices under non-trivial graph automorphisms. This paper focuses on studying the distinguishing number of inverse graphs, which are graphs associated with finite groups. We explore various properties and techniques related to finding the distinguishing number of inverse graphs. Specifically, we analyze the distinguishing number of inverse graphs associated with cyclic groups and provide exact values for their distinguishing numbers. Additionally, we present a general method for constructing distinguishing labels for inverse graphs associated with cyclic groups. The results presented in this paper contribute to the understanding of the distinguishing number of inverse graphs and offer insights into the symmetry of graphs associated with finite groups.

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