

Iterative criteria for oscillation of third-order linear delay dynamic equations

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ABSTRACT. We establish new oscillation criteria for a linear third-order functional dynamic equation on time scales. Our approach employs an iterative process combined with comparison principles to improve and generalize existing results. We demonstrate our findings through illustrative examples.

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REFERENCES

- [1] Adıvar, M.; Akin, E.; Higgins, R. Oscillatory behavior of solutions of third-order delay and advanced dynamic equations. *J. Inequal. Appl.* **2014** (2014), Paper No. 95, 1–16.
- [2] Agarwal, R. P.; Bohner, M. An oscillation criterion for first order delay dynamic equations. *Funct. Differ. Equ.* **16** (2009), 11–17.
- [3] Agarwal, R. P.; Bohner, M.; Li, T.; Zhang, C. Oscillation of third-order nonlinear delay differential equations. *Taiwanese J. Math.* **17** (2013), 545–558.
- [4] Alexander, R. D.; Moore, T. E. *The evolutionary relationships of 17-year and 13-year cicadas, and three new species (Homoptera, Cicadidae, Magicicada)*. Misc. Publ. Mus. Zool. Univ. Mich. **121** (1962), 1–57.
- [5] Alzabut, J.; Grace, S. R.; Santra, S. S.; Chhatria, G. N. Asymptotic and oscillatory behaviour of third order nonlinear differential equations with canonical operator and mixed neutral terms. *Qual. Theory Dyn. Syst.* **22** (2023), Paper No. 15, 1–17.
- [6] Berezhansky, L.; Domoshnitsky, A.; Koplatadze, R. *Oscillation, Nonoscillation, Stability and Asymptotic Properties for Second and Higher Order Functional Differential Equations*. Chapman and Hall/CRC, 2020.
- [7] Bohner, M.; Peterson, A. *Dynamic Equations on Time Scales: An Introduction with Applications*. Birkhäuser, Boston, 2001.
- [8] Bohner, M.; Peterson, A. *Advances in Dynamic Equations on Time Scales*. Birkhäuser, Boston, 2003.
- [9] Bohner, M. Some oscillation criteria for first order delay dynamic equations. *Far East J. Appl. Math.* **18** (2005), 289–304.
- [10] Chatzarakis, G. E.; Grace, S. R.; Jadlovská, I. Oscillation criteria for third-order delay differential equations. *J. Inequal. Appl.* **2017** (2017), Paper No. 330, 11 pp.
- [11] Elabbasy, E. M.; Hassan, T. S.; Elmatary, B. M. Oscillation criteria for third order delay nonlinear differential equations. *Electron. J. Qual. Theory Differ. Equ.* **2012** (2012), 1–11.
- [12] Gao, L.; Zhang, Q.; Liu, S. Oscillatory behavior of third-order nonlinear delay dynamic equations on time scales. *J. Comput. Appl. Math.* **256** (2014), 104–113.
- [13] Georgiev, S. G. *Functional Dynamic Equations on Time Scales*. Springer Nature, Switzerland AG, 2019.
- [14] Grace, S. R.; Chhatria, G. N. On oscillatory behaviour of third-order half-linear dynamic equations on time scales. *Opuscula Math.* **42** (2023), 849–865.
- [15] Grace, S. R.; Chhatria, G. N. Oscillation of higher order nonlinear dynamic equations with a nonlinear neutral term. *Math. Methods Appl. Sci.* **46** (2023), no. 2, 2373–2388.
- [16] Grace, S. R.; Graef, J. R.; Tunc, E. On the oscillation of certain third order nonlinear dynamic equations with a nonlinear damping term. *Math. Slovaca* **67** (2017), no. 2, 501–508.

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- [17] Grace, S. R.; Graef, J. R.; Tunc, E. Oscillatory behavior of a third-order neutral dynamic equation with distributed delays. *Electron. J. Qual. Theory Differ. Equ.* **2016** (2016), no. 14, 1–14.
- [18] Grace, S. R.; Jadlovská, I.; Chhatria, G. N. Oscillation criteria for general third-order delay dynamic equations. *Qual. Theory Dyn. Syst.* **24** (2025), 1–17.
- [19] Han, Z.; Li, T.; Sun, S.; Cao, F. Oscillation criteria for third order nonlinear delay dynamic equations on time scales. *Ann. Polon. Math.* **99** (2010), 143–156.
- [20] Han, Z.; Li, T.; Sun, S.; Zhang, M. Oscillation behavior of solutions of third-order nonlinear delay dynamic equations on time scales. *Commun. Korean Math. Soc.* **26** (2011), 499–513.
- [21] Hayat, T.; Wang, Y.; Siddiqui, A. M.; Hutter, K.; Asghar, S. Peristaltic transport of a third-order fluid in a circular cylindrical tube. *Math. Models Methods Appl. Sci.* **12** (2002), 1691–1706.
- [22] Huang, X.; Deng, X.; Wang, Q. The asymptotic behavior and oscillation for a class of third order nonlinear delay dynamic equations. *Acta Math. Sci. Ser. B (Engl. Ed.)* **44** (2024), no. 3, 925–946.
- [23] Hilger, S. Analysis on measure chains—a unified approach to continuous and discrete calculus. *Results Math.* **18** (1990), 18–56.
- [24] Karpuz, B. Sharp oscillation and nonoscillation tests for delay dynamic equations. *Math. Methods Appl. Sci.* **42** (2019), 2993–3001.
- [25] Li, T.; Han, Z.; Sun, S.; Zhao, Y. Oscillation results for third-order nonlinear delay dynamic equations on time scales. *Bull. Malays. Math. Sci. Soc.* **34** (2011), 639–648.
- [26] Mandelzweig, V. B.; Tabakin, F. Quasilinearization approach to nonlinear problems in physics with application to nonlinear ODEs. *Comput. Phys. Commun.* **141** (2001), 268–281.
- [27] Sabir, Z.; Said, S. B. Heuristic computing for the novel singular third order perturbed delay differential model arising in thermal explosion theory. *Arab. J. Chem.* **16** (2023), Art. 104509, 12 pp.
- [28] Saker, S. H. *Oscillation Theory of Delay Differential and Difference Equations: Second and Third Orders*. LAP Lambert Academic Publishing, 2010.
- [29] Saker, S. H.; Alzabut, J.; Mukheimer, A. On the oscillatory behavior for a certain class of third-order nonlinear delay difference equations. *Electron. J. Qual. Theory Differ. Equ.* **2010** (2010), no. 67, 1–16.
- [30] Saker, S. H. Oscillation of third-order functional dynamic equations on time scales. *Sci. China Math.* **54** (2011), 2597–2614.
- [31] Schlichting, H. *Boundary Layer Theory*. McGraw-Hill, New York, 1979.
- [32] Wang, Y.; Xu, Z. Asymptotic properties of solutions of certain third-order dynamic equations. *J. Comput. Appl. Math.* **236** (2012), 2354–2366.

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