CARPATHIAN J. MATH.
Volume **42** (2026), No. 1,
Pages 81-94

Online version at https://www.carpathian.cunbm.utcluj.ro/
Print Edition: ISSN 1584 - 2851; Online Edition: ISSN 1843 - 4401
DOI: https://doi.org/10.37193/CJM.2026.01.06

The Solodov–Svaiter type proximal point algorithm on a complete geodesic space

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ABSTRACT. Approximation of zeros of monotone operators can be applied to solve some nonlinear problems on Banach spaces such as function spaces. Similarly, a concept of monotone set-valued vector fields on geodesic spaces includes classes of convex minimisation problems and equilibrium problems. In this paper, we prove a zero point approximation theorem with a projection method for a monotone vector field on complete geodesic spaces. This method guarantees us to generate a sequence converging strongly to a zero point of a given set-valued vector field.

Acknowledgement. The author is grateful to anonymous referees for the comments and suggestions on the manuscript.

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Received: 12.02.2025. In revised form: 26.05.2025. Accepted: 11.09.2025
2020 *Mathematics Subject Classification.* 47H04, 47H05, 58C06.
Key words and phrases. *Geodesic space, monotone vector field, projection method, proximal point algorithm.*
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