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On Stability of Two New Generalized Set-Valued Functional Equations

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ABSTRACT. Denote by $C_c(Y)$ the set of all closed convex subsets of a Banach space Y. For every $A, B \in C_c(Y)$, we define an operation $\oplus$ by $A \oplus B := \overline{A+B}$ which is a closure of the set $A + B := \{a + b : a \in A,\ b \in B\}$. The present work aims to establish the Hyers-Ulam-Rassias stability of the following generalized set-valued functional equations

$$F\left(\frac{x+y}{2}+(\alpha-1)z\right) \oplus F\left(\frac{x+z}{2}+(\alpha-1)y\right) \oplus F\left(\frac{y+z}{2}+(\alpha-1)x\right) = \alpha\left(F(x) \oplus F(y) \oplus F(z)\right)$$

and

$$F(\beta x+y) \oplus F(\beta x-y) = F(x+y) \oplus F(x-y) \oplus 2(\beta^2-1)F(x),$$

for all $x, y, z \in X$, where $F : X \to C_c(Y)$ is an unknown set-valued function while X is a real vector space, $\alpha \geq 2$ and $\beta \notin \{-1, 0, 1\}$ are fixed integers. These two equations are respectively related to Cauchy-Jensen type and quadratic type set-valued functional equations.

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