# Amenable gyrogroups and their fixed-point property[*]

TEERAPONG SUKSUMRAN[1] AND JATURON WATTANAPAN[2,3]

ABSTRACT. An amenable group is a group that possesses a left-invariant mean and, furthermore, satisfies the Markov–Kakutani Fixed-Point Theorem (also known as Day's Fixed-Point Theorem). In this article, we examine gyrogroups that satisfy the Markov–Kakutani–Day fixed-point property, and also give a characterization of amenable groups related to a fixed-point property.

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[1] CHIANG MAI UNIVERSITY
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS
CHIANG MAI 50200, THAILAND
*Email address*: teerapong.suksumran@cmu.ac.th

[2] CHIANG MAI UNIVERSITY
FACULTY OF SCIENCE
CHIANG MAI 50200, THAILAND

[3] CHIANG MAI UNIVERSITY
OFFICE OF RESEARCH ADMINISTRATION
CHIANG MAI 50200, THAILAND
*Email address*: jaturon.w@cmu.ac.th