

A Double Inertial Fixed Point Algorithm with Linesearch and Its Application to Machine Learning for Data Classification

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ABSTRACT. In this paper, we introduce and study a new accelerated common fixed point algorithm based on the viscosity approximation, double inertial, and linesearch technique. The convergence properties and practical applications of the proposed algorithm are explored, highlighting its effectiveness in solving bilevel optimization problems and its potential in machine learning for data classification. Based on our experiment, it is found that our proposed algorithm has superior convergence behaviour than the existing algorithms in the literature.

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