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Solving Certain Classes of Variational Inclusion Problems Using Linearized Splitting Algorithms with Applications

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ABSTRACT. In this paper, we introduce numerical algorithms for finding a solution of certain classes of inclusion problems in Hilbert spaces. Under some appropriate assumptions, we established strong and weak convergence results for the sequences generated by the proposed algorithms. Moreover, we remark that by considering different cases, we can obtain several useful results on important classes of inclusion problems. Hence, our main results generalize and extend related results existing in the literature. Some theoretical applications of the general problem are also given. Finally, we validate the performance of one of our proposed algorithm by solving an image recovery problem and the numerical results show that our proposed algorithm performs well.

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