

Accelerated forward-backward algorithm based on inertial and correction terms with linesearch for solving convex minimization problem and its application

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ABSTRACT. This paper presents an accelerated variant of the proximal forward-backward splitting method designed for solving convex minimization problem in Hilbert spaces. Our proposed algorithm integrates an inertial extrapolation term and two additional correction terms, coupled with linesearch stepsize that circumvents the explicit need for Lipschitz constant estimation. We establish weak convergence theorem, demonstrating that our method approximates solutions to convex minimization problems. Numerical experiments confirm the practical effectiveness and accelerated convergence speed of our algorithm, particularly highlighting its application in image recovery problem.

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