

Double-step inertial algorithms for equilibrium and fixed point problems in Hilbert spaces

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ABSTRACT. In this paper, we introduce a novel double-inertial algorithm for simultaneously solving equilibrium problems and finding common fixed points of countable families of nonexpansive mappings in real Hilbert spaces. Under mild conditions on the inertial parameters and step sizes, we establish a weak convergence theorem for the proposed method. Furthermore, we provide numerical experiments that demonstrate the algorithm's convergence behavior and compare its performance with existing methods.

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REFERENCES

- [1] Blum, E.; Oettli, W. From optimization and variational inequalities to equilibrium problems. *Math. Stud.* **63** (1994), 123–145.
- [2] Browder, F. E. Fixed point theorems for noncompact mappings in Hilbert spaces. *Proc Nat Acad Sci USA.* **53** (1965), 1272–1276.
- [3] Bussaban, L.; Suantai, S.; Kaewkhao, A. A parallel inertial S-iteration forward-backward algorithm for regression and classification problems. *Carpathian J. Math.* **36** (2020), 35–44. <https://doi.org/10.37193/CJM.2020.01.04>
- [4] Ceng, L.C.; Yao, J.C. A hybrid iterative scheme for mixed equilibrium problems and fixed point problems. *J. Comput. Appl. Math.* **214** (2008), no. 1, 186–201.
- [5] Cholakijak, P. A hybrid iterative scheme for equilibrium problems, variational inequality problems, and fixed point problems in Banach spaces. *Fixed Point Theory Appl.* (2009), 1–18.
- [6] Cholakijak, P.; Abdou, A.A.; Cho, Y.J. Proximal point algorithms involving fixed points of nonexpansive mappings in CAT(0) spaces. *Fixed Point Theory Appl.* (2015), no. 1, 1–13.
- [7] Colao, V.; Marino, G. Strong convergence for a minimization problem on points of equilibrium and common fixed points of an infinite family of nonexpansive mappings. *Nonlinear Anal.* **73** (2010), no. 11, 3513–3524.
- [8] Combettes, P.L.; Hirstoaga, S. A. Equilibrium programming in Hilbert spaces. *J. Nonlinear Convex Anal.* **6** (2005), no. 1, 117–136.
- [9] He, Z. The split equilibrium problem and its convergence algorithms. *J. Inequal. Appl.* (2012), 162.
- [10] Iiduka, H. Convergence analysis of iterative methods for nonsmooth convex optimization over fixed point sets of quasi-nonexpansive mappings. *Math. Program.* **159** (2016), 509–538.
- [11] Inkrong, P.; Cholakijak, P. On multi-inertial extrapolations and forward-backward-forward algorithms. *Carpathian J. Math.* **40** (2024), no. 2, 293–305.
- [12] Izuchukwu, C.; Aphane, M.; Aremu, K. O. Two-step inertial forward-reflected-anchored-backward splitting algorithm for solving monotone inclusion problems. *Comput. Appl. Math.* **42** (2023), 351. <https://doi.org/10.1007/s40314-023-02485-6>
- [13] Iyiola, O. S.; Shehu, Y. Convergence results of two-step inertial proximal point algorithm. *Appl. Numer. Math.* **182** (2022), 57–75. <https://doi.org/10.1016/j.apnum.2022.07.013>

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- [14] Janngam, K.; Suantai, S.; Wattanataweekul, R. A novel fixed-point based two-step inertial algorithm for convex minimization in deep learning data classification. *AIMS Mathematics* **10** (2025), no. 3, 6209–6232.
- [15] Li, W.; Xiao, Y. B.; Wang, X.; Feng, J. Existence and stability for a generalized differential mixed quasi-variational inequality. *Carpathian J. Math.* **34** (2018), no. 3, 347–354.
- [16] Moudafi, A.; Al-Shemas, E. Simultaneous iterative methods for split equality problem. *Trans. Math. Program. Appl.* **1** (2013), 1–11.
- [17] Nakajo, K.; Shimoji, K.; Takahashi, W. Strong convergence to common fixed points of families of nonexpansive mappings in Banach spaces. *J. Nonlinear Convex Anal.* **8** (2007), 11–34.
- [18] Nakajo, K.; Shimoji, K.; Takahashi, W. On strong convergence by the hybrid method for families of mappings in Hilbert spaces. *Nonlinear Anal. Theor. Methods Appl.* **71** (2009), 112–119.
- [19] Nguyen, T.T.V.; Strodiot, J.J.; Nguyen, V.H. Hybrid methods for solving simultaneously an equilibrium problem and countably many fixed point problems in a Hilbert space. *J. Optim. Theory Appl.* **160** (2014), no. 3, 809–831.
- [20] Opial, Z. Weak convergence of the sequence of successive approximations for nonexpansive mappings. *Bull. Amer. Math. Soc.* **73** (1967), no. 4, 591–597. <https://doi.org/10.1090/S0002-9904-1967-11761-0>
- [21] Petrot, N.; Tangkhaiwetkul, J. The stability of dynamical system for the quasi mixed equilibrium problem in Hilbert spaces. *Thai J. Math.* **18** (2020), no. 3, 1433–1446.
- [22] Tada, A.; Takahashi, W. Strong convergence theorem for an equilibrium problem and a nonexpansive mapping. *Nonlinear analysis and convex analysis* (2007), 609–617.
- [23] Tan, K.; Xu, H. Approximating fixed points of nonexpansive mappings by the Ishikawa iteration process. *J. Math. Anal. Appl.* **178** (1993), 301–308.
- [24] Thong, D. V.; Reich, S.; Li, X. H.; Tham, P. T. H. An efficient algorithm with double inertial steps for solving split common fixed-point problems and an application to signal processing. *Comput. Appl. Math.* **44** (2025), 102. <https://doi.org/10.1007/s40314-024-03058-x>
- [25] Tiammee, J.; Suantai, S. On solving split best proximity point and equilibrium problems in Hilbert spaces. *Carpathian J. Math.* **35** (2019), no. 3, 385–392.
- [26] Tiammee, S.; Tiammee, J. Weak convergence of inertial proximal point algorithm for a family of nonexpansive mappings in Hilbert spaces. *Carpathian J. Math.* **40** (2024), no. 1, 173–185.

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