

Hölderian Stability for Parametric Optimization Models and Applications

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ABSTRACT. In this paper, we investigate the Hölderian stability of parametric optimization and parametric equilibrium problems using the upper bound function of the objective functions. This approach allows us to avoid imposing strong convexity or strong monotonicity conditions on the objective functions, which are commonly used assumptions that typically ensure the uniqueness of solutions in reference problems. Consequently, we successfully establish Hölderian stability for these problems even in cases where their solution sets are not necessarily singletons. As an application of our findings, we conduct a stability analysis of the Lancaster models. Our approach differs from existing studies in the literature, and the results we obtain are new.

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