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On Convex Contractions Based on the Measure of Noncompactness: Theory and Applications

ISHAK ALTUN, İLKER GENÇTÜRK, AND HATICE ASLAN HANÇER

ABSTRACT. In this paper, we establish that any continuous mapping satisfying the condition of convex contraction with respect to the measure of noncompactness has at least one fixed point within a bounded, closed, and convex subset of a Banach space. Our results significantly generalize the Darbo fixed-point theorem. To demonstrate the applicability of these theoretical results, we present an existence theorem for the solution of a specific integral equation under certain conditions. Finally, we illustrate the practical use of the existence theorem with a concrete example.

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Corresponding author: Ishak Altun; ialtun@kku.edu.tr

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Department of Mathematics, Faculty of Engineering and Natural Science, Kirikkale University, 71450 Yahsihan, Kirikkale, Turkey
Email address: ialtun@kku.edu.tr
Email address: ilkergencturk@gmail.com
Email address: haticeaslanhancer@gmail.com