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A New Direction-based Method for Solving Constrained Multiobjective Optimization Problems

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ABSTRACT. This paper introduces the Topkis-Veinott Multiobjective Optimization (TVMO) method for nonlinear constrained multiobjective problems (MOPs). TVMO extends the direction-based Topkis–Veinott approach from single-objective to multiobjective settings and, unlike scalarization methods with predefined weights, is parameter-free and works directly with the vector of objectives. It handles infeasibility and constraint violations explicitly rather than through penalties. Each iteration solves an auxiliary quadratic subproblem to obtain a feasible improving direction, followed by a line search that selects the step length to maintain feasibility and ensure sufficient decrease of all objectives. Numerical experiments show that TVMO effectively approximates the nondominated front and outperforms some existing methods across multiple criteria, indicating strong reliability for nonlinear constrained MOPs.

REFERENCES

[1] Ansary, M. A. T.; Panda, G. A sequential quadratically constrained quadratic programming technique for a multiobjective optimization problem. *Eng. Optim.* **51** (2019), no. 1, 22–41.
[2] Ansary, M. A. T.; Panda, G. A sequential quadratic programming method for constrained multiobjective optimization problems. *J. Comput. Appl. Math.* **64** (2020), no. 1, 379–397.
[3] Armijo, L. Minimization of functions having Lipschitz continuous first partial derivatives. *Pacific J. Math.* **16** (1966), no. 1, 1–3.
[4] Bandyopadhyay, S.; Pal, S. K.; Aruna, B. Multiobjective GAs, quantitative indices, and pattern classification. *IEEE Trans. Syst. Man Cybern. B Cybern.* **34** (2004), no. 5, 2088–2099.
[5] Bazaraa, M. S.; Sherali, H. D.; Shetty, C. M. *Nonlinear Programming: Theory and Algorithms*. John Wiley & Sons, Hoboken, NJ, 2006.
[6] Boriwan, P.; Kuroiwa, D.; Petrot, N. On the properties of lexicographic tolerable robust solution sets for uncertain multiobjective optimization problems. *Carpathian J. Math.* **37** (2021), no. 3, 417–426.
[7] Birge, J. R.; Qi, L.; Wei, Z. A variant of the Topkis-Veinott method for solving inequality constrained optimization problems. *Appl. Math. Optim.* **41** (2000), 309–330.
[8] Custódio, A. L.; Madeira, J. F.; Vaz, A. I.; Vicente, L. N. Direct multisearch for multiobjective optimization. *SIAM J. Optim.* **21** (2011), no. 4, 1109–1140.
[9] Carrizo, G. A.; Fazzio, N. S.; Schuverdt, M. L. A nonmonotone projected gradient method for multiobjective problems on convex sets. *J. Oper. Res. Soc. China* (2022), 1–18.
[10] Cocchi, G.; Lapucci, M. An augmented Lagrangian algorithm for multiobjective optimization. *Comput. Optim. Appl.* **77** (2020), no. 1, 29–56.
[11] Cocchi, G.; Lapucci, M.; Mansueto, P. Pareto front approximation through a multiobjective augmented Lagrangian method. *EURO J. Comput. Optim.* **9** (2021), 100008.
[12] Collette, Y.; Siarry, P. *Multiobjective Optimization: Principles and Case Studies*. Springer, Berlin, 2013.
[13] Deb, K. *Multiobjective Optimization Using Evolutionary Algorithms*. John Wiley & Sons, Chichester, 2001.
[14] Deb, K.; Pratap, A.; Meyarivan, T. Constrained test problems for multiobjective evolutionary optimization. *Evolutionary Multi-Criterion Optimization*, 284–298, Springer, Berlin, Heidelberg, 2001.

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[15] Deb, K.; Thiele, L.; Laumanns, M.; Zitzler, E. Scalable test problems for evolutionary multiobjective optimization. *Evolutionary Multiobjective Optimization: Theoretical Advances and Applications*, 197–236, Springer, London, 2005.
[16] Dolan, E. D.; Moré, J. J. Benchmarking optimization software with performance profiles. *Math. Program.* **91** (2002), 201–213.
[17] Dolatnezhadsomarin, A.; Khorram, E. Two efficient algorithms for constructing almost even approximations of the Pareto front in multiobjective optimization problems. *Eng. Optim.* **51** (2019), no. 4, 567–589.
[18] Ehrgott, M. *Multicriteria Optimization*. Springer, Berlin, 2005.
[19] Eichfelder, G. *Adaptive Scalarization Methods in Multiobjective Optimization*. Springer, Berlin, 2008.
[20] Fliege, J.; Graña Drummond, L. M.; Svaiter, B. F. Newton's method for multiobjective optimization. *SIAM J. Optim.* **20** (2009), no. 2, 602–626.
[21] Fliege, J.; Svaiter, B. F. Steepest descent methods for multicriteria optimization. *Math. Methods Oper. Res.* **51** (2000), no. 3, 479–494.
[22] Fliege, J.; Vaz, A. I. F. A method for constrained multiobjective optimization based on SQP techniques. *SIAM J. Optim.* **26** (2016), no. 4, 2091–2119.
[23] Fukuda, E. H.; Drummond, L. G.; Raupp, F. M. A barrier-type method for multiobjective optimization. *Optimization* **69** (2020), no. 11, 2471–2487.
[24] Fukuda, E. H.; Drummond, L. M. G.; Raupp, F. M. P. An external penalty-type method for multicriteria optimization. *TOP* **24** (2016), no. 3, 493–513.
[25] Ghaznavi-Ghosoni, B. A.; Khorram, E. On approximating weakly/properly efficient points in multiobjective programming. *Math. Comput. Model.* **54** (2011), no. 11, 3172–3181.
[26] Gebken, B.; Peitz, S.; Dellnitz, M. A descent method for equality and inequality constrained multiobjective optimization problems. *Numerical and Evolutionary Optimization – NEO 2017*, 29–61. Springer, Cham, 2019.
[27] Ghalavand, N.; Khorram, E.; Morovati, V. An adaptive nonmonotone line search for multiobjective optimization problems. *Comput. Oper. Res.* **136** (2021), 105506.
[28] Graña Drummond, L. M. A projected gradient method for vector optimization problems. *Comput. Optim. Appl.* **28** (2004), no. 1, 5–29.
[29] Graña Drummond, L. M.; Svaiter, B. F. A steepest descent method for vector optimization. *J. Comput. Appl. Math.* **175** (2005), no. 2, 395–414.
[30] Hwang, C. L.; Masud, A. S. M. *Multiple Objective Decision Making: Methods and Applications*. Lecture Notes in Economics and Mathematical Systems, 164. Springer, Berlin, 1979.
[31] Ivanov, V. I. Optimality conditions for an isolated minimum of order two in C^1 constrained optimization. *J. Math. Anal. Appl.* **356** (2009), no. 1, 30–41.
[32] Jiang, R.; Deng, Y.; Chen, Y.; Luo, H.; An, B. Deep reinforcement learning for multiobjective game strategy selection. *Comput. Oper. Res.* **168** (2024), 106683.
[33] Khaledian, K.; Khorram, E.; Soleimani-damaneh, M. Strongly proper efficient solutions: efficient solutions with bounded trade-offs. *J. Optim. Theory Appl.* **168** (2016), no. 3, 864–883.
[34] Köbis, E.; Tammer, C. Robust vector optimization with a variable domination structure. *Carpathian J. Math.* **33** (2017), no. 3, 343–351.
[35] Knowles, J.; Thiele, L.; Zitzler, E. *A Tutorial on the Performance Assessment of Stochastic Multiobjective Optimizers*. TIK Report 214, Computer Engineering and Networks Laboratory, ETH Zurich, Zurich, 2006.
[36] Maeda, T. Constraint qualifications in multiobjective optimization problems: differentiable case. *J. Optim. Theory Appl.* **80** (1994), no. 3, 483–500.
[37] Miettinen, K. *Nonlinear Multiobjective Optimization*. Springer, New York, 2012.
[38] Miholca, M. Vector variational-like inequalities and vector optimization problems. *Carpathian J. Math.* **30** (2014), no. 1, 101–108.
[39] Morovati, V.; Pourkarimi, L. Extension of Zoutendijk method for solving constrained multiobjective optimization problems. *Eur. J. Oper. Res.* **273** (2019), no. 1, 44–57.
[40] Morovati, V.; Pourkarimi, L.; Basirzadeh, H. Barzilai and Borwein's method for multiobjective optimization problems. *Numer. Algorithms* **72** (2016), no. 3, 539–604.
[41] Pirouz, B.; Khorram, E. A computational approach based on the ε-constraint method in multiobjective optimization problems. *J. Stat. Adv. Theory Appl.* **49** (2016), 453-483.
[42] Povalej, Z. Quasi-Newton's method for multiobjective optimization. *J. Comput. Appl. Math.* **255** (2014), 765–777.
[43] Qu, S.; Ji, Y.; Jiang, J.; Zhang, Q. Nonmonotone gradient methods for vector optimization with a portfolio optimization application. *Eur. J. Oper. Res.* **263** (2017), no. 2, 356–366.
[44] Rashidi, M.; Soleimani-damaneh, M. MultiSQP-GS: a sequential quadratic programming algorithm via gradient sampling for nonsmooth constrained multiobjective optimization. *Comput. Optim. Appl.* (2024), 1–39.

[45] Rashidi, M.; Khorram, E.; Soleimani-damaneh, M. An exact penalty method with nonmonotone line search and rapid infeasibility detection for constrained multiobjective optimization: Application in supervised machine learning. *Comput. Oper. Res.* **188** (2026), 107351.
[46] Shahbeyk, S.; Soleimani-damaneh, M. Limiting proper minimal points of nonconvex sets in finite-dimensional spaces. *Carpathian J. Math.* **35** (2019), no. 3, 379–384.
[47] Topkis, D. M.; Veinott, A. F. On the convergence of some feasible direction algorithms for nonlinear programming. *SIAM J. Control* **5** (1967), no. 2, 268–279.
[48] Yahaya, J.; Kumam, P.; Abubakar, J. Efficient nonlinear conjugate gradient techniques for vector optimization problems. *Carpathian J. Math.* **40** (2024), no. 2, 515–533.
[49] Zitzler, E.; Thiele, L.; Laumanns, M.; Fonseca, C. M.; da Fonseca, V. G. Performance assessment of multiobjective optimizers: an analysis and review. *IEEE Trans. Evol. Comput.* **7** (2003), no. 2, 117–132.

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