

On a sequence of integral operators associated with power Lindley distribution

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ABSTRACT. The aim of this paper is to introduce a new sequence of probabilistic-type integral operators based on the power Lindley distribution. We study their approximation properties in spaces of weighted continuous functions, providing estimates for the rate of convergence in terms of various moduli of smoothness, as well as an asymptotic formula. Moreover, several illustrative examples and comparisons are presented. Finally, we consider a generalization of the power Lindley distribution and compute some important related quantities, such as the information potential, expected value, and variance. The paper ends with an application of the proposed integral operators to the derivation of bounds for the information potential.

ACKNOWLEDGEMENTS

The authors acknowledge the support for the project “Tech4You–Technologies for climate change adaptation and quality of life improvement”, Mission 4, Component 2, Investment 1.5, funded by the Next Generation EU—Italian National Recovery and Resilience Plan (NRRP). The paper has been performed within the activities of the Italian GNAMPA-INdAM Group, the UMI Group TAA “Approximation Theory and Applications” and, of the network RITA (Research Italian network on Approximation).

This study was funded by the Next Generation EU—Italian National Recovery and Resilience Plan (NRRP), Mission 4, Component 2, Investment 1.5, project “Tech4You–Technologies for climate change adaptation and quality of life improvement”, number ECS0000009 – CUP C43C22000400006.

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Received: 22.10.2025. In revised form: 22.03.2026. Accepted: 31.03.2026

2020 *Mathematics Subject Classification.* 41A25, 41A35, 41A36, 41A81, 60E05, 47A58.

Key words and phrases. *Approximation by positive linear operators, asymptotic formula, rate of convergence, probability distribution.*

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