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New classes of compact-type spaces

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ABSTRACT. Being motivated by the notions of κ-Fréchet–Urysohn spaces and k'-spaces introduced by Arhangel'skii, the notion of sequential spaces and the study of Ascoli spaces, we introduce three new classes of compact-type spaces. They are defined by the possibility to attain each or some of boundary points x of an open set U by a sequence in U converging to x or by a relatively compact subset $A \subseteq U$ such that $x \in \overline{A}$. Relationships of the introduced classes with the classical classes (as, for example, the classes of κ-Fréchet–Urysohn spaces, (sequentially) Ascoli spaces, $k_{\mathbb{R}}$-spaces, $s_{\mathbb{R}}$-spaces etc.) are given. We characterize these new classes of spaces and study them with respect to taking products, subspaces and quotients. In particular, we give new characterizations of κ-Fréchet–Urysohn spaces and show that each feathered topological group is κ-Fréchet–Urysohn. We describe locally compact abelian groups which endowed with the Bohr topology belong to one of the aforementioned classes. Numerous examples are given.

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REFERENCES

[1] Arhangel'skii, A.V. Bicompact sets and topology of spaces. (Russian) *Dokl. Akad. Nauk SSSR* **150** (1963), 9–12.
[2] Arhangel'skii, A.V. Some types of factor mappings and the relations between classes of topological spaces. (Russian) *Dokl. Akad. Nauk SSSR* **153** (1963), 743–746.
[3] Arhangel'skii, A.V. *Topological function spaces*. *Math. Appl.* **78**, Kluwer Academic Publishers, Dordrecht, 1992.
[4] Arhangel'skii, A.V.; Tkachenko, M.G. *Topological groups and related strutures*. Atlantis Press/World Scientific, Amsterdam-Raris, 2008.
[5] Bagley, R.W.; Weddington, D.D. Products of k'-spaces. *Proc. Amer. Math. Soc.* **22** (1969), 392–394.
[6] Banakh, T.; Gabriyelyan, S. On the C_k-stable closure of the class of (separable) metrizable spaces. *Monatsh. Math.* **180** (2016), 39–64.
[7] Engelking, R. *General Topology.* Heldermann Verlag, Berlin, 1989.
[8] Franklin, S.P.; Smith Thomas, B.V. A survey on k_ω-spaces. *Topology Proc.* **2** (1977), 111–124.
[9] Gabriyelyan, Saak. Free locally convex spaces and the k-space property. *Canadian Math. Bull.* **57** (2014), 803–809.
[10] Gabriyelyan, Saak. Topological properties of spaces of Baire functions. *J. Math. Anal. Appl.* **478** (2019), 1085–1099.
[11] Gabriyelyan, Saak. Locally convex properties of free locally convex spaces. *J. Math. Anal. Appl.* **480** (2019), Paper No. 123453, 16 pp.
[12] Gabriyelyan, Saak. Ascoli and sequentially Ascoli spaces. *Topology Appl.* **285** (2020), Paper No. 107401, 28 pp.
[13] Gabriyelyan, Saak. Ascoli's theorem for pseudocompact spaces. *Revista de la Real Academia de Ciencias Exactas, Fisicas y Naturales. Serie A. Matematicas RACSAM* **114** (2020), Paper No. 174, 10 pp.

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[14] Gabriyelyan, Saak. Pełczyński-type sets and Pełczyński's geometrical properties of locally convex spaces. *Dissertationes Math.* **605** (2025), 133 pp.
[15] Gabriyelyan, S.; Grebík, J.; Kąkol, J.; Zdomskyy, L. The Ascoli property for function spaces. *Topology Appl.* **214** (2016), 35–50.
[16] Gabriyelyan, S.; Kąkol, J.; Plebanek, G. The Ascoli property for function spaces and the weak topology of Banach and Fréchet spaces. *Studia Math.* **233** (2016), 119–139.
[17] Gabriyelyan, S.; Reznichenko, E. On $k_{\mathbb{R}}$-spaces and $s_{\mathbb{R}}$-spaces. *Topol. Appl.* **373** (2025), Paper No. 109528, 24 pp.
[18] Gabriyelyan, S.; Reznichenko, E. Functions on products $X \times Y$ with applications to Ascoli spaces, $k_{\mathbb{R}}$-spaces and $s_{\mathbb{R}}$-spaces. *European J. Math.* **11** (2025), Paper No. 72, 18pp.
[19] Gruenhage, G.; Tanaka, Y. Products of k-spaces and spaces of countable tightness. *Proc. Amer. Math. Soc.* **273**, 299–308.
[20] Hewitt, E.; Ross, K.A. *Abstract Harmonic Analysis*. Vol. I. 2nd ed. Springer-Verlag, Berlin, 1979.
[21] Jarchow, H. *Locally Convex Spaces*. B.G. Teubner, Stuttgart, 1981.
[22] Just, W.; Weese, M. *Discovering Modern Set Theory II*, Graduate Studies in Mathematics. American Math. Soc. **18**, 1997.
[23] Lin, F.; Lin, S.; Liu, C. The k_R-property of free Abelian topological groups and products of sequential fans. *Topol. Appl.* **240** (2018), 78–97.
[24] Liu, C.; Ludwig, L.D. κ-Fréchet–Urysohn spaces. *Houston J. Math.* **31** (2005), 391–401.
[25] McCoy, R.A.; Ntantu, I. *Topological Properties of Spaces of Continuous Functions*. Lecture Notes in Math. **1315**, 1988.
[26] Michael, E. Local compactness and cartesian products of quotient maps and k-spaces. *Ann. Inst. Fourier, Grenoble* **18**:2 (1968), 281–286.
[27] Michael, E. A quintuple quotient quest. *General Topology and Appl.* **2** (1972), 91–138.
[28] Michael, E. On k-spaces, k_R-spaces and $k(X)$. *Pacific J. Math.* **47** (1973), 487–498.
[29] Mrówka, S. Mazur theorem and m-adic spaces. *Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys.* 18 (1970), 299–305.
[30] Noble, N. Ascoli theorems and the exponential map. *Trans. Amer. Math. Soc.* **143** (1969), 393–411.
[31] Noble, N. The continuity of functions on Cartesian products. *Trans. Amer. Math. Soc.* **149** (1970), 187–198.
[32] Nyikos, P.J. Metrizability and Fréchet-Urysohn property in topological groups. *Proc. Amer. Math. Soc.* **83** (1981), 793–801.
[33] Sakai, M. Two properties of $C_p(X)$ weaker than Fréchet–Urysohn property. *Topology Appl.* **153** (2006), 2795–2804.
[34] Sakai, M. κ-Fréchet-Urysohn property of $C_k(X)$. *Topology Appl.* **154** (2007), 1516–1520.
[35] Shchepin, E. On κ-metrizable spaces. *Math. USSR, Izv.* **14** (1980), 1–34.
[36] Schlüchtermann, G.; Wheeler, R.F. The Mackey dual of a Banach space. *Noti de Matematica* **XI** (1991), 273–287.
[37] Simon, P. A compact Fréchet space whose square is not Fréchet. *Comment. Math. Univ. Carolin.* **21**:4 (1980), 749–753.

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