CARPATHIAN J. MATH.
Volume **42** (2026), No. 4,
Pages 853-869

Online version at https://www.carpathian.cunbm.utcluj.ro/
Print Edition: ISSN 1584 - 2851; Online Edition: ISSN 1843 - 4401
DOI: https://doi.org/10.37193/CJM.2026.04.08

On local monotonic convergence analysis of the IVPs of nonlinear Caputo fractional ordinary hybrid differential equations

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ABSTRACT. We prove some approximation results concerning the existence and uniqueness of the local solution of initial value problems of nonlinear Caputo fractional hybrid differential equations by using the monotone iteration method given by Dhage (2014) which is based on the recent hybrid fixed point principles. Next, an approximation result for Ulam-Hyers stability of the local solution of the considered equation is established and the monotonic dependence of the local solution on initial data is also discussed. Finally, a couple of examples are indicated to illustrate the hypotheses and abstract results of this paper.

Acknowledgment. The authors are thankful to Professor S. K. Ntouyas (Greece) and the referees for giving some useful suggestions for the improvement of this paper.

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Received: 27.10.2025. In revised form: 12.02.2026. Accepted: 02.04.2026
2010 *Mathematics Subject Classification.* 34A45, 34B15, 47H07, 47H10.
Key words and phrases. *Fractional differential equation; Hybrid fixed point principle; Monotone iteration method; Local approximation theorem, Ulam-Hyers stability, Monotonic dependence on initial data.*
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