

# Well-posedness and scalarization for set-valued equilibrium problems

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**ABSTRACT.** This paper aims to study the well-posedness of set-valued equilibrium problems. First, some notions of global well-posedness and well-setness for a set-valued equilibrium problem are introduced. Further, we compare them and relationships among these concepts are established. Also, we give sufficient conditions on the data in order to guarantee global well-posedness and well-setness. Main results are applied to Browder variational inclusions. Moreover, we relate these notions with the stability of appropriate scalar optimization problems, using the generalized oriented distance function. Second, we have introduced and investigated the notion of well-posedness under perturbation in terms of convergent sequences of objective maps. Also, in generalized convex settings, sufficient conditions for sequentially well-posedness are given. Finally, we provide examples to illustrate our concepts and results.

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Received: 08.11.2025. In revised form: 12.02.2026. Accepted: 15.04.2026

2020 *Mathematics Subject Classification.* 49K40 · 90C31.

Key words and phrases. *Set-valued equilibrium problem · Global well-posedness · Well-setness · Generalized oriented distance · Browder variational inclusion · Sequentially well-posedness.*

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