

On the dynamics of a family of max-type difference equations

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ABSTRACT. We fully determine the dynamics of the max-type difference equation $x_{n+1} = \frac{\max\{x_n, A\}}{x_n x_{n-1}}$, where A and the initial conditions are real positive numbers, thereby completing the partial results existing in the literature. We achieve this by applying a different approach from the one used by other authors, based now on a topological conjugacy that transforms the max-type equation into a piecewise linear one. Finally, we outline some further lines of research related to the topic.

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