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Approximation of Solutions to Variational Inequalities via Halpern-Type Iterations with Error Terms

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ABSTRACT. We introduce the notion of a *quasi-Halpern sequence* and establish necessary and sufficient conditions for its weak and strong convergence in Hilbert spaces. The proposed framework extends the classical Halpern iteration by incorporating summable or asymptotically vanishing computational errors. As an application, we employ the result to obtain approximate solutions of variational inequality problems.

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