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Fixed-Time Stability and Noise Robustness of a Polynomial-Gain Gradient Neural Network for Absolute Value Equations

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ABSTRACT. In this paper, we propose a Fixed-time Gradient Neural Network (FGNN) for solving the absolute value equation $Ax - |x| = b$. The FGNN employs a polynomial-gain architecture that combines sub-linear, super-linear, and reciprocal damping terms. We prove global fixed-time stability of the unperturbed system with an explicit settling-time bound and establish four robustness theorems: fixed-time stability and fixed-time boundedness under both bounded vanishing and bounded non-vanishing noise. Numerical experiments across nine models spanning three reformulation families validate FGNN's fixed-time convergence, dual noise robustness, and practical efficacy. A limitation analysis of the exponential predefined-time gain reveals catastrophic floating-point overflow at moderate initial residuals, further motivating the polynomial-gain design as a numerically reliable alternative with rigorous theoretical guarantees.

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REFERENCES

[1] Bhat, S. P.; Bernstein, D. S. Finite-time stability of continuous autonomous systems. *SIAM J. Control Optim.* **38** (2000), no. 3, 751–766.
[2] Caccetta, L.; Qu, B.; Zhou, G. A globally and quadratically convergent method for absolute value equations. *Compu. Optim. Appl.* **48** (2011), 45–58.
[3] Cao, M.; Xiao, L.; Zuo, Q.; Tan, P.; He, Y.; Gao, X. A fixed-time robust ZNN model with adaptive parameters for redundancy resolution of manipulators. *IEEE Trans. Emerg. Topics Comput. Intell.* **8** (2024), 3886–3898.
[4] Chen, C.; Yang, Y.; Yu, D.; Han, D. An inverse-free dynamical system for solving the absolute value equations. *Appl. Numer. Math.* **168** (2021), 170–181.
[5] Han, X.; He, X.; Ju, X.; Chen, J. Unified single-layer inverse-free neurodynamic network for solving absolute value equations. *IEEE Trans. Circuits Syst. II: Express Briefs* **71** (2024), 1166–1170.
[6] Hu, C.; Yu, J.; Chen, Z.; Jiang, H.; Huang, T. Fixed-time stability of dynamical systems and fixed-time synchronization of coupled discontinuous neural networks. *Neural Networks* **89** (2017), 74–83.
[7] Hu, D.; He, X.; Ju, X. A modified projection neural network with fixed-time convergence. *Neurocomputing* **489** (2022), 90–97.
[8] Jia, L.; Xiao, L.; Dai, J.; Wang, Y. Intensive noise-tolerant zeroing neural network based on a novel fuzzy control approach. *IEEE Trans. Fuzzy Syst.* **31** (2023), 4350–4360.
[9] Jin, L.; Su, Z.; Fu, D.; Xiao, X. Coevolutionary neural solution for nonconvex optimization with noise tolerance. *IEEE Trans. Neural Netw. Learn. Syst.* **35** (2023), 17571–17581.
[10] Ju, X.; Li, C.; Han, X.; He, X. Neurodynamic network for absolute value equations: A fixed-time convergence technique. *IEEE Trans. Circuits Syst. II: Express Briefs* **69** (2022), no. 3, 1807–1811.

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[11] Ju, X.; Yang, X.; Feng, G.; Che, H. Neurodynamic optimization approaches with finite/fixed-time convergence for absolute value equations. *Neural Networks* **165** (2023), 971–981.
[12] Ju, X.; Yuan, S.; Yang, X.; Shi, P. Fixed-time neurodynamic optimization algorithms and application to circuits design. *IEEE Trans. Circuits Syst. I: Reg. Papers* **71** (2024), 2171–2181.
[13] Li, W.; Guo, C.; Ma, X.; Pan, Y. A strictly predefined-time convergent and noise-tolerant neural model for solving linear equations with robotic applications. *IEEE Trans. Ind. Electron.* **71** (2024), 798–809.
[14] Mangasarian, O. L.; Meyer, R. R. Absolute value equations. *Linear Algebra Appl.* **419** (2006), no. 2–3, 359–367.
[15] Mangasarian, O. L. Linear complementarity as absolute value equation solution. *Optim. Lett.* **8** (2014), no. 4, 1529–1534.
[16] Mansoori, A.; Erfanian, M. A dynamic model to solve the absolute value equations. *J. Comput. Appl. Math.* **333** (2018), 28–35.
[17] Mansoori, A.; Eshaghnezhad, M.; Effati, S. An efficient neural network model for solving the absolute value equations. *IEEE Trans. Circuits Syst. II: Express Briefs* **65** (2018), no. 3, 391–395.
[18] Polyakov, A. Nonlinear feedback design for fixed-time stabilization of linear control systems. *IEEE Trans. Automat. Control* **57** (2012), no. 8, 2106–2110.
[19] Prathom, K.; Wangkeeree, R.; Belay, Y. A.; Hongsri, A. A projection neural network with delays and optimization approaches for solving absolute value equations. *IEEE Access* **14** (2026), 50141–50155.
[20] Wangkeeree, R.; Wangkeeree, R.; Nazemi A.; Belay, Y.A.; Ungchittrakool, K. A projection recurrent neural network method for solving absolute value equations associated with second-order cones. J. Comput. Appl. Math. **485**(2026),117567.
[21] Wangkeeree, R.; Wangkeeree, R.; Belay, Y.A.; Ungchittrakool, K.; Nazemi, A. Neurodynamical neural network method for solving support vector machines with epigraph reformulated generalized pinball loss function. J. Comput. Appl. Math. **487**(2026).
[22] Rodríguez, E. J.; Vázquez, A. J. M.; Torres, J. D. S.; Defoort, M.; Loukianov, A. G. A Lyapunov-like characterization of predefined-time stability. *IEEE Trans. Automat. Control* **65** (2020), no. 11, 4922–4927.
[23] Saheya, B.; Nguyen, C. T.; Chen, J.-S. Neural network based on systematically generated smoothing functions for absolute value equation. *J. Appl. Math. Comput.* **61** (2019), no. 1, 533–558.
[24] Tan, Z.; Yang, Z. Robustness analysis of a fixed-time convergent GNN for online solving Sylvester equation and its application to robot path tracking. *IEEE Access* **11** (2023), 101873–101880.
[25] Wang, D.; Liu, X. A varying-parameter fixed-time gradient-based dynamic network for convex optimization. *Neural Networks* **167** (2023), 798–809.
[26] Wei, L.; Jin, L. Collaborative neural solution for time-varying nonconvex optimization with noise rejection. *IEEE Trans. Emerg. Topics Comput. Intell.* **8** (2024), 2935–2948.
[27] Yang, X.; Ju, X.; Shi, P.; Wen, G. Two novel noise-suppression projection neural networks with fixed-time convergence for variational inequalities and applications. *IEEE Trans. Neural Netw. Learn. Syst.* **36** (2023), 1707–1718.
[28] Yong, L. Iteration method for absolute value equation and applications in two-point boundary value problem of linear differential equation. *J. Inter. Math.* **18** (2015), no. 4, 355–374.
[29] Yu, D.; Zhang, G.; Chen, C.; Han, D. The neural network models with delays for solving absolute value equations. *Neurocomputing 589* (2023), 127707.
[30] Yu, D.; Zhang, G.; Zhang, T. Predefined-time robust adaptive gradient neural network for solving linear time-varying equations and its applications. *Expert Syst. Appl.* **255** (2024), 124546.
[31] Yu, D.; Zhang, G.; Ma, T. Neurodynamic approaches for solving absolute value equations and circuit implementation. *Chaos Solitons Fractals* (2025), 115714.
[32] Yu, D.; Ma, T.; Zhang, G.; Chen, C.; Han, D. Predefined-time robust gradient neural network for solving absolute value equations. *J. Comput. Appl. Math.* **473** (2026), 116887.
[33] Zhang, M.; Zheng, B. Accelerating noise-tolerant zeroing neural network with fixed-time convergence to solve the time-varying Sylvester equation. *Automatica* **135** (2022), 109998.
[34] Zhang, X.; Li, C.; Zhang, L.; Hu, Y.; Peng, Z. Convergence-accelerated fixed-time dynamical methods for absolute value equations. *J. Optim. Theory Appl.* **203** (2024), 600–628.
[35] Zhang, Y.; Liao, B.; Geng, G. GNN model with robust finite-time convergence for time-varying systems of linear equations. *IEEE Trans. Syst. Man Cybern. Syst.* **54** (2024), 4786–4797.
[36] Zuo, Q.; Li, K.; Xiao, L. Robust finite-time zeroing neural networks with fixed and varying parameters for solving dynamic generalized Lyapunov equation. *IEEE Trans. Neural Netw. Learn. Syst.* **33** (2021), 7695–7705.

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