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An upper bound on summation between disjoint cross intersecting families of permutations

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ABSTRACT. For natural numbers $\lambda \geq n$, let $[\lambda] = \{1, 2, ..., \lambda\}$. Denote by $\binom{[\lambda]}{n}$ the family of sets of n natural numbers $\{A \subseteq [\lambda] : |A| = n\}$ and $\mathcal{P}[\lambda, n]$ the family of permutations formed by all ordered pairs $\{(1, a_1), (2, a_2), ..., (n, a_n)\}$ such that $\{a_1, a_2, ..., a_n\} \in \binom{[\lambda]}{n}$. For a natural number t, two families $\mathcal{A}, \mathcal{B} \subseteq \mathcal{P}[\lambda, n]$ are said to be *cross t-intersecting* if $|A \cap B| \geq t$ for all $A \in \mathcal{A}$ and $B \in \mathcal{B}$, and are *disjoint* if $\mathcal{A} \cap \mathcal{B} = \emptyset$. Existing approaches to bounding $|\mathcal{A}| + |\mathcal{B}|$ and $|\mathcal{A}| \cdot |\mathcal{B}|$ for cross t-intersecting families over the past 20 years typically rely on spectral methods or translation operators. In this paper, we present a new approach that applies the chromatic polynomial of $(3 \times n)$-Latin rectangles, whose entries are taken from $[\lambda]$ for $\lambda \geq n$. By employing mathematical induction and a suitable coloring scheme, we derive an upper bound on $|\mathcal{A}| + |\mathcal{B}|$ when $\mathcal{A}$ and $\mathcal{B}$ are disjoint cross t-intersecting families.

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