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Best Ulam constant for a linear difference equation

ALINA-RAMONA BAIAS, FLORINA BLAGA and DORIAN POPA

ABSTRACT. In this paper we provide some results on Ulam stability for the linear difference equation of order one in Banach spaces and we determine its best Ulam constant. The main result is applied to a process of loan amortization.

1. INTRODUCTION

The starting point of stability theory for functional equations was the problem of S. M. Ulam concerning approximate homomorphisms of groups, formulated in 1940 during a talk at Madison University, Wisconsin (see [24]).

The first answer to Ulam's problem was given a year later by D. H. Hyers who proved that Cauchy's equation in Banach spaces is stable. Generally, we say that an equation is stable in Ulam sense if for every approximate solution of it there exists an exact solution of the equation near it. For more details and results on Ulam stability we refer the reader to [1, 6, 13, 5].

The problem of Ulam stability for difference equations is connected to the notion of perturbation of a discrete dynamical system. Results on Ulam stability of linear difference equation were obtained by J. Brzdek, D. Popa and B. Xu [6, 8, 18, 19].

The best constant in Ulam stability was studied in [12, 23] where is given a characterization of Ulam stability for linear operators and a representation of their best constant. The best constant for some classical operators in approximation theory was obtained by D. Popa and I. Raşa in [20, 21, 22]. J. Brzdek, S. M. Jung and M. Th Rassias obtained sharp estimates for the Ulam constant of some linear difference equations of the second order [7, 14]. As far as we know the best Ulam constant for the linear difference equation with complex coefficients was not provided yet, a result on the best constant for a first order difference equation with real coefficients was obtained by M. Onitsuka in [17].

The goal of this paper is to give some new results on Ulam stability for the linear difference equation of order one in Banach spaces and to determine its best Ulam constant.

2. MAIN RESULTS

Let $\mathbb{K}$ be one of the fields $\mathbb{R}$ or $\mathbb{C}$ and $(X, \|\cdot\|)$ a Banach space over $\mathbb{K}$. By $\mathbb{N}$ we denote the set of all nonnegative integers. We deal with Ulam stability of the linear difference equation (recurrence)

(2.1) $$x_{n+1} = ax_n + b_n,\ n \in \mathbb{N},\ x_0 \in X,$$

where $a \in \mathbb{K}$, $a \neq 0$, is a constant and $(b_n)_{n\geq 0}$ is a sequence in X.

Lemma 2.1. *If $(x_n)_{n\geq 0}$ satisfies the linear difference equation (2.1) then*

(2.2) $$x_n = a^n x_0 + b_0 a^{n-1} + b_1 a^{n-2} + ... + b_{n-2} a + b_{n-1}, n \geq 1.$$

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Corresponding author: Dorian Popa; Popa.Dorian@math.utcluj.ro.

Proof. Induction. □

Definition 2.1. The equation (2.1) is called stable in Ulam sense if there exists a constant $L \geq 0$ such that for every $\varepsilon > 0$ and every $(x_n)_{n\geq 0}$ in X satisfying

$$\|x_{n+1} - ax_n - b_n\| \leq \varepsilon, \ \ n \geq 0, \tag{2.3}$$

there exists a sequence $(y_n)_{n\geq 0}$ in X with the proprieties

$$y_{n+1} = ay_n + b_n, \ n \geq 0, \tag{2.4}$$

and

$$\|x_n - y_n\| \leq L\varepsilon, \ n \geq 0. \tag{2.5}$$

A sequence $(x_n)_{n\geq 0}$ which satisfies (2.3) for some $\varepsilon > 0$ is called an approximate solution of the linear difference equation (2.1). So, we can reformulate Definition 2.1 as folows: the equation (2.1) is called Ulam stable if for every approximate solution of it there exists an exact solution close to it.

If in Definition 2.1 the number ε is replaced by a sequence of positive numbers $(\varepsilon_n)_{n\geq 0}$ and $L\varepsilon$ from (2.5) by a sequence of positive numbers $(\delta_n)_{n\geq 0}$ the equation (2.1) is called generalized stable in Ulam sense.

The number L from (2.5) is called an Ulam constant of the equation (2.1). In what follows, we will denote by L_R the infimum of all Ulam constants of (2.1). If L_R is an Ulam constant for (2.1), then we call it *the best Ulam constant* or *the Ulam constant* of the equation (2.1). In general the infimum of all Ulam constants of an equation is not an Ulam constant of that equation (see [12, 21]).

In this paper we obtain a result on generalized Ulam stability of the equation (2.1) and consequently we determine its best Ulam constant.

The next result improves the estimates between the approximate solution and the exact solution for the equation (2.1) given in [18, 19].

Theorem 2.1. *Let $(\varepsilon_n)_{n\geq 0}$ be a sequence of nonnegatives numbers, $a \in \mathbb{K}$, $a \neq 0$, and suppose that the series $\sum\limits_{n=0}^{\infty} \frac{\varepsilon_n}{|a|^n}$ is convergent. Then for every sequence $(x_n)_{n\geq 0}$ in X with the property*

$$\|x_{n+1} - ax_n - b_n\| \leq \varepsilon_n, \ n \geq 0, \tag{2.6}$$

there exists a unique sequence $(y_n)_{n\geq 0}$ satisfying

$$y_{n+1} = ay_n + b_n, \ n \geq 0, \tag{2.7}$$

and

$$\|x_n - y_n\| \leq \sum_{k=0}^{\infty} \frac{\varepsilon_{n+k}}{|a|^{k+1}}, \ n \geq 1. \tag{2.8}$$

Proof. **Existence**. Suppose that $(x_n)_{n\geq 0}$ is a sequence in X satisfying relation (2.6) and let

$$x_{n+1} - ax_n - b_n =: c_n, \ \ n \geq 0.$$

Then

$$\|c_n\| \leq \varepsilon_n, n \geq 0,$$

and according the Lemma 2.1, we get

$$x_n = a^n \left(x_0 + \sum_{k=1}^{n} \frac{b_{k-1} + c_{k-1}}{a^k} \right), \ n \geq 1.$$

Consider the series

$$\sum_{n=1}^{\infty} \frac{c_{n-1}}{a^n}.$$

Since

$$\|\frac{c_{n-1}}{a^n}\| \leq \frac{\varepsilon_{n-1}}{|a|^n}, \; n \geq 1,$$

and the series

$$\sum_{n=1}^{\infty} \frac{\varepsilon_{n-1}}{\|a\|^n}$$

is convergent it follows, in view of the first comparison test, that the series

$$\sum_{n=1}^{\infty} \frac{c_{n-1}}{a^n}$$

is absolutely convergent. Let

$$\sum_{n=1}^{\infty} \frac{c_{n-1}}{a^n} = s, \; s \in X.$$

Define now the sequence $(y_n)_{n\geq 0}$ by the recurrence

$$y_{n+1} = ay_n + b_n, \; y_0 = x_0 + s, n \geq 0.$$

Then, in view of Lemma 2.1, we get

$$y_n = a^n \left(y_0 + \sum_{k=1}^{n} \frac{b_{k-1}}{a^k} \right), \; n \geq 1.$$

Consequently,

$$\begin{aligned} x_n - y_n &= a^n \left(x_0 - y_0 + \sum_{k=1}^{n} \frac{c_{k-1}}{a^k} \right) = \\ &= a^n \left(-s + \sum_{k=1}^{n} \frac{c_{k-1}}{a^k} \right) \\ &= -a^n \sum_{k=0}^{\infty} \frac{c_{n+k}}{a^{n+k+1}}, n \geq 0, \end{aligned}$$

hence

$$\begin{aligned} \|x_n - y_n\| &= |a|^n \left\| \sum_{k=0}^{\infty} \frac{c_{n+k}}{a^{n+k+1}} \right\| \leq |a|^n \sum_{k=0}^{\infty} \frac{\varepsilon_{n+k}}{|a|^{k+1+n}} \\ &= \sum_{k=0}^{\infty} \frac{\varepsilon_{n+k}}{|a|^{k+1}}, \; n \geq 0, \end{aligned}$$

and the existence is proved.

Uniqueness. Suppose that for a sequence $(x_n)_{n\geq 0}$ in X satisfying (2.6) there exist two sequences $(y_n)_{n\geq 0}$, $(z_n)_{n\geq 0}$ satisfying the equation (2.1) and

$$\|x_n - y_n\| \leq \sum_{k=0}^{\infty} \frac{\varepsilon_{n+k}}{|a|^{k+1}}, n \geq 0,$$

$$\|x_n - z_n\| \leq \sum_{k=0}^{\infty} \frac{\varepsilon_{n+k}}{|a|^{k+1}}, n \geq 0.$$

Then

$$\|y_n - z_n\| \leq \|y_n - x_n\| + \|x_n - z_n\| \leq 2\sum_{k=0}^{\infty} \frac{\varepsilon_{n+k}}{|a|^{k+1}}, n \geq 0.$$

Now, taking account of Lemma 2.1, we get

$$\|a^n(y_0 - z_0)\| \leq 2\sum_{k=0}^{\infty} \frac{\varepsilon_{n+k}}{|a|^{k+1}},$$

or

$$\|y_0 - z_0\| \leq \frac{2}{|a|}\sum_{k=0}^{\infty} \frac{\varepsilon_{n+k}}{|a|^{n+k}},\ n \geq 0. \tag{2.9}$$

But

$$R_n = \sum_{n=0}^{\infty} \frac{\varepsilon_{n+k}}{|a|^{n+k}},\ n \geq 0,$$

is the remainder of order $(n-1)$ of the convergent series

$$\sum_{n=0}^{\infty} \frac{c_n}{|a|^n},$$

therefore

$$\lim_{n\to\infty} R_n = 0.$$

Letting now $n \to \infty$ in (2.9), we get $y_0 = z_0$, therefore $y_n = z_n$, for every $n \geq 0$, hence the uniqueness is proved. □

As a consequence, a results on classical Ulam stability of equation (2.1) is given below. See also [18].

Corollary 2.1. *Suppose that $|a| \neq 1$ and let $\varepsilon > 0$. Then for every sequence $(x_n)_{n\geq 0}$ in X with the property*

$$\|x_{n+1} - ax_n - b_n\| \leq \varepsilon,\ n \geq 0, \tag{2.10}$$

there exists a sequence $(y_n)_{n\geq 0}$ such that

$$y_{n+1} = ay_n + b_n,\ n \geq 0, \tag{2.11}$$

$$\|x_n - y_n\| \leq \frac{\varepsilon}{||a| - 1|},\ n \geq 0. \tag{2.12}$$

Moreover, if $|a| > 1$ the sequence $(y_n)_{n\geq 0}$ is unique.

Proof. Let $|a| > 1$ and take $\varepsilon_n = \varepsilon$, $n \geq 0$, in Theorem 2.1. Then the series

$$\sum_{n=0}^{\infty} \frac{\varepsilon_n}{|a|^n} = \varepsilon\sum_{k=0}^{\infty} \frac{1}{|a|^n} = \frac{\varepsilon|a|}{|a| - 1}$$

is convergent and

$$\sum_{k=0}^{\infty} \frac{\varepsilon_{n+k}}{|a|^{k+1}} = \varepsilon\sum_{k=0}^{\infty} \frac{1}{|a|^{k+1}} = \frac{\varepsilon}{|a| - 1}.$$

Hence the conclusion of the theorem holds with uniqueness for $(y_n)_{n\geq 0}$ and the Ulam constant

$$L = \frac{1}{|a| - 1},$$

in view of Theorem 2.1.

Let now $|a| < 1$ and let $(x_n)_{n\geq 0}$ be a sequence in X which satisfies (2.10). Let the sequence $(c_n)_{n\geq 0}$ be given by

$$x_{n+1} - ax_n - b_n =: c_n, n \geq 0.$$

Then, according to Lemma 2.1, we get

$$x_n = a^n x_0 + (b_0 + c_0)a^{n-1} + ... + (b_{n-2} + c_{n-2})a + b_{n-1} + c_{n-1}, n \geq 1.$$

Define the sequence $(y_n)_{n\geq 0}$ by

$$y_{n+1} = ay_n + b_n,\ n \geq 0,\ y_0 = x_0.$$

Then

$$\begin{aligned} \|x_n - y_n\| &= \left\|c_0 a^{n-1} + ... + c_{n-2}a + c_{n-1}\right\| \\ &\leq \|c_0\| \, |a|^{n-1} + ... + \|c_{n-2}\| \, |a| + \|c_{n-1}\| \\ &\leq \varepsilon(1 + |a| + ... + |a|^n) \leq \frac{\varepsilon}{1 - |a|}, n \geq 1. \end{aligned}$$

□

Corollary 2.2. *Let $(\varepsilon_n)_{n\geq 0}$ be a sequence of positive numbers, $a \in \mathbb{K}$, $a \neq 0$, and suppose that*

$$\limsup \frac{\varepsilon_{n+1}}{|a| \, \varepsilon_n} < 1.$$

Then for every q and a with

$$\limsup \frac{\varepsilon_{n+1}}{|a| \, \varepsilon_n} < q < 1$$

and $|a| \, q < 1$ there exist $n_0 \in \mathbb{N}$ such that for every sequence $(x_n)_{n\geq 0}$ with the property

$$\|x_{n+1} - ax_n - b_n\| \leq \varepsilon_n,\ n \geq 0,$$

there exists a unique $(y_n)_{n\geq 0}$ satisfying the relations

$$y_{n+1} = ay_n + b_n, n \geq 0$$

$$\|x_n - y_n\| \leq \frac{\varepsilon_{n_0}}{|a| \, (1 - q)}, n \geq n_0.$$

Proof. Let

$$u_n = \frac{\varepsilon_n}{|a|^n}, n \geq 0.$$

Then

$$\limsup \frac{u_{n+1}}{u_n} = \limsup \frac{\varepsilon_{n+1}}{|a| \, \varepsilon_n} < 1,$$

so the series

$$\sum_{n=0}^{\infty} \frac{\varepsilon_n}{|a|^n}$$

is convergent. Then, according to Theorem 2.1 it follows that for every sequence $(x_n)_{n\geq 0}$ satisfying (2.6) there exists a unique sequence $(y_n)_{n\geq 0}$ such that relations (2.7) and (2.8) are fulfilled.

Now let q be such that

$$\limsup \frac{\varepsilon_{n+1}}{|a| \, \varepsilon_n} < q < 1, |a|q < 1.$$

Then there exists $n_0 \in \mathbb{N}$ such that

$$\frac{\varepsilon_{n+1}}{|a| \, \varepsilon_n} \leq q,$$

for all $n \geq n_0$. This relation leads to

$$\varepsilon_n \leq |qa|^{n-n_0} \cdot \varepsilon_{n_0},\ n \geq n_0.$$

We get

$$\begin{aligned} \sum_{k=0}^{\infty} \frac{\varepsilon_{n+k}}{|a|^{k+1}} &\leq \sum_{k=0}^{\infty} \frac{\varepsilon_{n_0} |qa|^{n+k-n_0}}{|a|^{k+1}} \\ &= \frac{\varepsilon_{n_0}}{|a|} |aq|^{n-n_0} (1 + q + q^2 + ...) \\ &= \frac{\varepsilon_{n_0}}{|a|} |aq|^{n-n_0} \cdot \frac{1}{1-q} \\ &\leq \frac{\varepsilon_{n_0}}{|a|} \cdot \frac{1}{1-q},\ n \geq n_0. \end{aligned}$$

□

Remark 2.1. In the theory of difference equations there exists also the notion of stability of an equilibrium point. We will discuss here, the relation between this type of stability and Ulam stability for the linear difference equation

$$x_{n+1} = ax_n + b,\ \ n \geq 0, \tag{2.13}$$

where $a \in \mathbb{K}$, $b \in X$, $x_0 \in X$.

An equilibrium point of the equation (2.13) is an element $x^* \in X$ with the property that $x^* = ax^* + b$, i.e., x^* is a fixed point of the function $f : X \to X$, $f(x) = ax + b$.

The equilibrium point x^* of the equation (2.13) is called *asymptotically stable* (or *attracting*) if there exists $\delta > 0$ such that for all $x_0 \in X$ with the property $\|x_0 - x^*\| < \delta$ it follows $\lim\limits_{n\to\infty} x_n = x^*$.

If $\delta = +\infty$, then x^* is called *globally asymptotically stable* or x^* is said to be a *global attractor*. For more details on the stability of an equilibrium point we refer the reader to [10, Chapter 4].

According to [10, Theorem 4.13] the equilibrium point x^* is asymptotically stable if $|a| < 1$ and asymptotically unstable if $|a| > 1$.

Then we can conclude that for the linear difference equation with constant coefficients the asymptotic stability of an equilibrium point is equivalent with the Ulam stability of the equation only for the case $|a| < 1$, while for the case $|a| > 1$ the equation (2.13) is Ulam stable and asymptotically unstable. Remark also that the notion of Ulam stability concerns the stability of the equation while asymptotic stability concerns the solution of the equation (the equilibrium point).

Theorem 2.2. *The best Ulam constant of the equation* (2.1) *is*

$$L_R = \frac{1}{||a| - 1|},\ a \in \mathbb{K},\ |a| \neq 1.$$

Proof. Suppose that equation (2.1) admits an Ulam constant $L < L_R$.

Let $\varepsilon > 0$, $u \in X$, $\|u\| = 1$, and let $(x_n)_{n\geq 0}$ be given by

$$x_{n+1} - ax_n - b_n = \varepsilon \cdot \frac{a^{n+1}}{|a|^{n+1}} \cdot u,\ n \geq 0.$$

Then

$$\|x_{n+1} - ax_n - b_n\| = \varepsilon,\ n \geq 0,$$

so there exist $(y_n)_{n\geq 0}$, satisfying $y_{n+1} = ay_n + b_n$, $n \geq 0$, such that

$$\|x_n - y_n\| \leq L\varepsilon,\ n \geq 0. \tag{2.14}$$

In view of Lemma 2.1 we obtain

$$x_n = a^n \left(x_0 + \sum_{k=1}^{n} \frac{b_{k-1} + \varepsilon \frac{a^k}{|a|^k} u}{a^k} \right), n \geq 1,$$

and

$$y_n = a^n \left(y_0 + \sum_{k=1}^{n} \frac{b_{k-1}}{a^k} \right), n \geq 1.$$

Then

$$\begin{aligned} x_n - y_n &= a^n \left(x_0 - y_0 + \varepsilon u \sum_{k=1}^{n} \frac{1}{|a|^k} \right) \\ &= a^n \left(x_0 - y_0 + \varepsilon u \cdot \frac{|a|^n - 1}{|a|^n (|a| - 1)} \right), n \geq 1. \end{aligned}$$

1^0. Let $|a| > 1$. We have

$$\lim_{n \to \infty} \left(x_0 - y_0 + \varepsilon u \frac{|a|^n - 1}{|a|^n (|a| - 1)} \right) = x_0 - y_0 + \frac{\varepsilon u}{|a| - 1}.$$

If

$$x_0 - y_0 + \frac{\varepsilon u}{|a| - 1} \neq 0,$$

it follows

$$\lim_{n \to \infty} \|x_n - y_n\| = +\infty,$$

a contradiction with relation (2.14).

If

$$x_0 - y_0 + \frac{\varepsilon u}{|a| - 1} = 0,$$

then it follows

$$x_n - y_n = -\frac{a^n \varepsilon u}{|a|^n (|a| - 1)}, n \geq 1,$$

so

$$\|x_n - y_n\| = \frac{\varepsilon}{|a| - 1} = \varepsilon L_R > \varepsilon L,$$

a contradiction with (2.14).

2^0. Let $|a| < 1$. Then

$$\begin{aligned} \|x_n - y_n\| &= \left\| a^n \left(x_0 - y_0 + \varepsilon u \frac{|a|^n - 1}{|a|^n (|a| - 1)} \right) \right\| \\ &= |a|^n \left\| x_0 - y_0 + \varepsilon u \frac{|a|^n - 1}{|a|^n (|a| - 1)} \right\| \\ &= \left\| |a|^n \left(x_0 - y_0 + \varepsilon u \frac{|a|^n - 1}{|a| - 1} \right) \right\|, \end{aligned}$$

therefore

$$\lim_{b \to \infty} \|x_n - y_n\| = \frac{\varepsilon}{1 - |a|} = \frac{\varepsilon}{||a| - 1|} = \varepsilon L_R,$$

a contradiction with (2.14).

□

3. Loan Amortization via linear difference equations

It is well known that a lot of economical and financial processes can be described by means of recursive rules between two consecutive elements, which from a mathematical point of view constitute a difference equations of the first order. This includes for example simple and compound interest calculation and loan amortization. In the sequel we will present a practical application of the results obtained in this paper in loan amortization. Suppose that a loan is repaid by a sequence of periodic payments. We denote by p_n the outstanding principal after the n-th payment q_n, $n \in \mathbb{N}$. We denote by r the interest rate. Then the sequence $(p_n)_{n\geq 0}$ satisfies the following difference equation (see[10])

$$p_{n+1} = (1+r)p_n - q_n,\ 0 \leq n \leq n_1, \tag{3.15}$$

where $p(0) = p_0$ is the initial debt. Then p_n is given by the following relation

$$p_n = (1+r)^n p_0 - \sum_{k=0}^{n-1} (1+r)^{n-k-1} q_k,\ 1 \leq n \leq n_1. \tag{3.16}$$

Let us suppose now that we want to change the scheme of payment of the outstanding principal. This will imply obviously a change of the periodic payments q_n, consequently, it is essential to know which are the differences between the two schemes of payments and how this affects the client. On the other hand, it is reasonable to assume that the periodic payments q_n is restricted (due to some exterior conditions), which implies $Q_n = q_n + t_n$, $n \geq 0$ with $|t_n| \leq \varepsilon$, $n \geq 0$, where Q_n denotes the n-th payment in the new scheme of amortization, with the same rate. Therefore for the new scheme we have

$$P_{n+1} = (1+r)P_n - q_n - t_n,\ 0 \leq n \leq n_2, \tag{3.17}$$

where $|t_n| \leq \varepsilon$ for some positive fixed ε, where P_n denotes the outstanding principal after the n-th payment. The relation (3.17) is equivalent to

$$|P_{n+1} - (1+r)P_n - q_n| \leq \varepsilon, 0 \leq n \leq n_2. \tag{3.18}$$

Now, according to Corollary 2.1 it follows that for every P_n satisfying (3.18) there exists p_n such that

$$p_{n+1} = (1+r)p_n - q_n,\ 0 \leq n \leq n_1$$

and

$$|P_n - p_n| \leq \frac{\varepsilon}{r},\ 0 \leq n \leq \min\{n_1, n_2\}. \tag{3.19}$$

The process is in agreement with reality if n is sufficiently large, for instance for loans for real estate purposes.

3.1. **An illustrative example.** Consider the case of home loans which are traditionally 15-year to 30-year fixed rate mortgages. Most people do not keep a loan for that long, they refinance the loan at some point or simply they choose the change the amortization schedule either by adding extra amounts of money to their monthly payments or by paying less, according with their financial situation. Any change of the amortization scheme will imply obviously a change of the outstanding principal or a change of the length of the loan period.

Example 3.1. As follows we will analyze the amortization schedule for $60,000$ EUR thirty year loan charging 10% interest. This will lead to a monthly payment of $q_n = 527$ EUR. Now supposing that the client pays slightly more, i.e. $Q_n = 533$ EUR monthly, this will lead to a reduction of the loan period with 2 years. In the below table we present a reduced form of both amortization schemes.

q_n	Interest	Principal	Balance	Q_n	Interest	Principal	Balance
q_1	500 E	27 E	59,973 E	Q_1	500 E	33 E	59,967 E
q_2	500 E	27 E	59,947 E	Q_2	500 E	33E	59,934 E
q_3	500 E	27 E	59,920 E	Q_3	499 E	34 E	59,901 E
...	...	...	...	...	...	...	....
q_{120}	455 E	72 E	54,491 E	Q_{120}	436 E	97 E	52,171 E
q_{240}	334 E	193 E	59,947 E	Q_{240}	295 E	238 E	35,111 E
...	...	...	...	...	...	...	....
q_{334}	106 E	421 E	12,261 E	Q_{334}	13 E	520 E	1,052 E
q_{335}	102 E	424 E	11,839 E	Q_{335}	9 E	524 E	528 E

Comparing the principals from the two monthly payments for the common periods, according to relation (3.19) it can be seen that the difference between the two principals remains less then 100 EUR, which is in agreement with our stability result for $\varepsilon = 10$ and $r = \dfrac{1}{10}$. In order to evaluate the maximum difference of the principals in two distinct amortization schemes ε can be chose by the client.

Since our stability results hold for infinitely many values of n, the behavior of the model on finite periods of time can influence the accuracy of the results.

References

[1] Agarwal, R. P., Xu, B. and Zhang, W., *Stability of functional equations in single variable*, J. Math. Anal. Appl., **288** (2003), 852–869

[2] Atalan, Y. and Karakaya, V., *Stability of nonlinear Volterra-Fredholm integro differential equation: a fixed point approach*, Creat. Math. Inform., **26** (2017), No. 3, 247–254

[3] Bojor, F., *On the stability of quartic type functional equation*, Creat. Math. Inform., **17** (2008), No. 3, 319–325 (2009)

[4] Bojor, F., *Generalized additive Cauchy equations and their Ulam-Hyers stability*, Creat. Math. Inform., **18** (2009), No. 2, 129–135

[5] Brillouët-Belluot, N., Brzdek, J. and Cieplinski, K., *On some recent developments in Ulam's type stability*, Abstr. Appl. Anal., (2012), 41 pp.

[6] Brzdek, J., Popa, D., Raşa, I. and Xu, B., *Ulam stability of Operators*, Academic Press, 2018

[7] Brzdek, J. and Jung, S. M., *A note on stability of an operator linear equation of the second order*, Abstr. Appl. Anal., (2011), Article ID 602713, 15 pp.

[8] Brzdek, J., Popa, D. and Xu, B., *The Hyers-Ulam stability of nonlinear recurrences*, J. Math. Anal. Appl., **335** (2007), 443–449

[9] Dumitrescu, S., Monea, M. and Mortici, C., *A survey on the stability of mean value points and functional equations involving some special functions*, Creat. Math. Inform., **24** (2015), No. 1, 27–42

[10] Elaydi, S., *An introduction to difference equations*, Springer, 2005

[11] Guran, L., *Ulam-Hyers stability of fixed point equations for singlevalued operators on KST spaces*, Creat. Math. Inform., **21** (2012), No. 1, 41–47

[12] Hatori, O., Kobayashi, K., Miura, T., Takagi, H. and Takahasi, S. E., *On the best constant of Hyers-Ulam stability*, J. Nonlinear Convex Anal., **5** (2004), 387–393

[13] Hyers, D. H., Isac, G. and Rassias, Th. M., *Stability of Functional Equations in Several Variables*, Birkhäuser Boston, Inc., 1998

[14] Jung, S. M. and Rassias, Th. M., *A linear functional equation of third order associated to the Fibonacci numbers*, Art. ID 137468, 7 pp., 2014

[15] Lazăr, V. L., *Ulam-Hyers stability for partial differential equations*, Creat. Math. Inform., **21** (2012), No. 1, 73–78

[16] Mleşniţe, O. M., *Ulam-Hyers stability for operatorial inclusions* Creat. Math. Inform., **21** (2012), No. 1, 87–94

[17] Onitsuka, M., *Hyers-Ulam stability of first-order nonhomogeneous linear difference equations with a constant step-size*, Appl. Math. Comput., **330** (2018), No. 1, 143–151

[18] Popa, D., *Hyers-Ulam stability of the linear recurrence with constant coefficients*, Adv. Difference Equ., **2** (2005), 101–107

[19] Popa, D., *Hyers-Ulam-Rassias stability of a linear recurrence*, J. Math. Anal. Appl., **309** (2005), No. 2, 591–597

[20] Popa, D. and Raşa, I., *On the stability of some classical operators from approximation theory*, Expo. Math., **31** (2013), 205–214

[21] Popa, D. and Raşa, I., *Best constant in stability of some positive linear operators*, Aequationes Math., **90** (2016), 719–726

[22] Popa, D. and Raşa, I., *On the best constant in Hyers-Ulam stability of some positive linear operators*, J. Math. Anal. Appl., **412** (2014), 103–108

[23] Takagi, H., Miura, T. and Takahasi, S. E., *Essential norms and stability constants of weighted composition operators on C(X)*, Bull. Korean Math. Soc., **40**, (2003), 583–591

[24] Ulam, S. M., *A Collection of Mathematical Problems*, Interscience, New York, 1960

DEPARTMENT OF MATHEMATICS
TECHNICAL UNIVERSITY OF CLUJ-NAPOCA
G. BARIŢIU 25, 400027 CLUJ-NAPOCA, ROMANIA
Email address: Baias.Alina@math.utcluj.ro.
Email address: Blaga.Florina@math.utcluj.ro.
Email address: Popa.Dorian@math.utcluj.ro.