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Dedicated to Prof. Juan Nieto on the occasion of his 60^{th} anniversary

Ulam-Hyers-Rassias stability of some quasilinear partial differential equations of first order

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ABSTRACT. In this paper we investigate the Ulam-Hyers-Rassias stability for some quasilinear partial differential equations.

1. INTRODUCTION

The Ulam stability is an important concept in the theory of functional equations. The origin of Ulam stability theory was a talk, given at Wisconsin University, in 1940, by S. M. Ulam [25], who formulated the following problem: We are given a group G_1 and a metric group G_2 with metric d. Given $\varepsilon > 0$, does there exist a $\delta > 0$ such that if $f : G_1 \to G_2$ satisfies

$$d\left(f\left(xy\right), f\left(x\right)f\left(y\right)\right) \leq \delta, x, y \in G_{1,}$$

then a homomorphism $g : G_1 \to G_2$ exists with

$$d\left(f\left(x\right), g\left(x\right)\right) \leq \varepsilon, x \in G_1?$$

The first partial answer to Ulam's question came within a year, when Hyers [7] proved the following result, for additive Cauchy equation in Banach spaces.

Let E_1, E_2 be Banach spaces and let $f : E_1 \to E_2$ be a transformation such that, for some $\delta > 0$,

$$\left\|f\left(x+y\right) - f\left(x\right) - f\left(y\right)\right\| \leq \delta$$

for all $x, y \in E_1$. There exists a unique additive mapping $g : E_1 \to E_2$ satisfying

$$\left\|f\left(x\right) - g\left(x\right)\right\| \leq \delta, \forall x \in E_1.$$

After Hyers' result a great number of papers on this subject have been published generalizing Hyers' theorem in many direction (see. e.g. [2, 3, 4, 5, 8, 14, 20, 21, 26, 22]. Alsina and Ger were the first authors who investigated the Ulam-Hyers stability of a differential equations ([1]).

They have proved that for every differentiable mapping $f : I \to \mathbb{R}$ satisying

$$\left|f'\left(x\right) - f\left(x\right)\right| \leq \varepsilon, \forall x \in I,$$

where $\varepsilon > 0$ is a given number and I is an open interval of $\mathbb{R}$, there exists a differentiable mapping $g : I \to \mathbb{R}$ such that $g'\left(x\right) = g\left(x\right)$ and

$$\left|f\left(x\right) - g\left(x\right)\right| \leq 3\varepsilon, \forall x \in I.$$

The result of Alsina and Ger was extended by Miura, Miyajima, Takahasi, Takagi and Jung [9, 10, 11, 19, 23, 24] to the stability of the first order linear differential equation and

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linear differential equations of higher order with constant coefficients. The study of Ulam-Hyers stability of partial differential equations started recently and we will mention here the results obtained in this direction by Jung [12, 13], Lungu and Ciplea [15], Lungu and Popa [16, 17], Lungu and Rus [18]. In [3] Brzdek, Popa, Rasa and Xu presented a unified and systematic approach to the field.

In what follows let $D = [a, b) \times \mathbb{R}, a \in \mathbb{R}, b \in \mathbb{R}$ be a subset of $\mathbb{R}^2$. Let $n \neq -1, 0$.

We deal with the Ulam-Hyers-Rassias stability of the quasilinear partial differential equation

$$p(x,y)\,u^n(x,y)\frac{\partial u}{\partial x} + q(x,y)\,u^n(x,y)\frac{\partial u}{\partial y} = r(x,y)\,u^{n+1}(x,y) + f(x,y), \tag{1.1}$$

$$u(a,y) = \psi(y) \tag{1.2}$$

where $p, q, r \in C(D,\mathbb{R}), f \in C(D,\mathbb{R})$ are given functions and $u \in C^1(D,\mathbb{R})$ is the unknown function. We suppose that $p(x,y) \neq 0$ for every $(x,y) \in D$.

We suppose that there exists $L > 0$ such that

$$\left|\frac{f(x,y)}{p(x,y)} \cdot \frac{1}{u^n(x,y)} - \frac{f(x,y)}{p(x,y)} \cdot \frac{1}{w^n(x,y)}\right| \leq L\,|u(x,y) - w(x,y)|, \tag{1.3}$$

for every $(x,y) \in D$ and $u, w \in C^1(D,\mathbb{R})$.

Definition 1.1. The equation (1.1) is Ulam-Hyers-Rassias stable with respect to $\phi \in C(D, R_+)$ if there exists $c_{f,\phi} > 0$ such that for each $\varepsilon > 0$ and for each solution $u \in C^1(D,\mathbb{R})$ of

$$\left|p(x,y)\,u^n(x,y)\frac{\partial u}{\partial x} + q(x,y)\,u^n(x,y)\frac{\partial u}{\partial y} - r(x,y)\,u^{n+1}(x,y) - f(x,y)\right| \leq \varepsilon\phi(x,y)$$

with the initial condition (1.2), there exists a solution $w \in C^1(D,\mathbb{R})$ of (1.1) with

$$|u(x,y) - w(x,y)| \leq c_{f,\phi}\varepsilon\phi(x,y), \forall (x,y) \in D.$$

2. Main results

We consider the characteristic system corresponding to quasilinear partial differential equation (1.1)

$$\frac{dx}{p \cdot u^n} = \frac{dy}{q \cdot u^n} = \frac{du}{r \cdot u^{n+1} + f}.$$

From the first equality we have

$$\frac{dx}{p(x,y)} = \frac{dy}{q(x,y)}$$

and hence

$$\frac{dy}{dx} = \frac{q(x,y)}{p(x,y)}. \tag{2.4}$$

Let $\varphi : [a,b) \to R$ be a solution of the above equation (2.4). Let

$$\phi(x,y) = e^{\int_a^x \frac{r(\theta,\varphi(\theta)+y-\varphi(x))}{p(\theta,\varphi(\theta)+y-\varphi(x))}d\theta}. \tag{2.5}$$

We study the Ulam-Hyers-Rassias stability for the equation (1.1), with initial condition (1.2), with respect to function ϕ from (2.5).

The main result of this paper is given in the next theorem.

Theorem 2.1. *If $\frac{r(x,y)}{p(x,y)} \leq M < 0$, for every $(x,y) \in D$ and $\widetilde{\phi}(s,t)$ is nondecreasing in s then the equation (1.1), with initial condition (1.2), is Ulam-Hyers-Rassias stable with respect to ϕ.*

Proof. We consider the change of coordinates $(x,y) \to (s,t)$

$$\begin{cases} x = s \\ y = t + \varphi(s) \end{cases}.$$

Define the function $\widetilde{u}$ by

$$\widetilde{u}(s,t) = u(s, \varphi(s) + t) \Leftrightarrow u(x,y) = \widetilde{u}(x, y - \varphi(x)).$$

and the function $\widetilde{\phi}$ by

$$\widetilde{\phi}(s,t) = \phi(s, \varphi(s) + t) \Leftrightarrow \phi(x,y) = \widetilde{\phi}(x, y - \varphi(x)).$$

We also define $\widetilde{p}(s,t) = p(s, \varphi(s) + t)$, $\widetilde{q}(s,t) = q(s, \varphi(s) + t)$, $\widetilde{r}(s,t) = r(s, \varphi(s) + t)$, $\widetilde{f}(s,t) = f(s, \varphi(s) + t)$ and $\widetilde{\psi}(t) = \psi(\varphi(a) + t)$.

Hence

$$\widetilde{\phi}(s,t) = e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)} d\theta}. \tag{2.6}$$

Then

$$\begin{cases} \frac{\partial u}{\partial x} = \frac{\partial \widetilde{u}}{\partial s} - \varphi'(s) \cdot \frac{\partial \widetilde{u}}{\partial t} \\ \frac{\partial u}{\partial y} = \frac{\partial \widetilde{u}}{\partial t} \end{cases}$$

and replacing in (1.1) it follows

$$\widetilde{p}\widetilde{u}^n \frac{\partial \widetilde{u}}{\partial s} - p\widetilde{u}^n \varphi'(s) \cdot \frac{\partial \widetilde{u}}{\partial t} + \widetilde{q}\widetilde{u}^n \frac{\partial \widetilde{u}}{\partial t} = \widetilde{r}\widetilde{u}^{n+1} + \widetilde{f},$$

or

$$\widetilde{p}\widetilde{u}^n \frac{\partial \widetilde{u}}{\partial s} + \widetilde{u}^n \left[\widetilde{q} - \widetilde{p}\varphi'(s)\right] \cdot \frac{\partial \widetilde{u}}{\partial t} = \widetilde{r}\widetilde{u}^{n+1} + \widetilde{f}.$$

Since $\widetilde{q} - \widetilde{p}\varphi'(s) = 0$ we have

$$\widetilde{p}\widetilde{u}^n \frac{\partial \widetilde{u}}{\partial s} - \widetilde{r}\widetilde{u}^{n+1} = \widetilde{f},$$

or

$$\frac{\partial \widetilde{u}}{\partial s}(s,t) - \frac{\widetilde{r}(s,t)}{\widetilde{p}(s,t)} \cdot \widetilde{u}(s,t) = \frac{\widetilde{f}(s,t)}{\widetilde{p}(s,t)} \cdot \frac{1}{\widetilde{u}^n(s,t)}. \tag{2.7}$$

$$\widetilde{u}(a,t) = \widetilde{\psi}(t) \tag{2.8}$$

We study the Ulam-Hyers-Rassias stability for the equation (2.7) with initial condition (2.8), with respect to function $\widetilde{\phi}$ from (2.6). Let $\varepsilon > 0$ and $\widetilde{u}(s,t)$ be an approximate solution of the above problem. Consider the inequality

$$-\varepsilon\widetilde{\phi}(s,t) \le \frac{\partial \widetilde{u}}{\partial s}(s,t) - \frac{\widetilde{r}(s,t)}{\widetilde{p}(s,t)} \cdot \widetilde{u}(s,t) - \frac{\widetilde{f}(s,t)}{\widetilde{p}(s,t)} \cdot \frac{1}{\widetilde{u}^n(s,t)} \le \varepsilon\widetilde{\phi}(s,t)$$

We have

$$-\varepsilon e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)} d\theta} \le \frac{\partial \widetilde{u}}{\partial s}(s,t) - \frac{\widetilde{r}(s,t)}{\widetilde{p}(s,t)} \cdot \widetilde{u}(s,t) - \frac{\widetilde{f}(s,t)}{\widetilde{p}(s,t)} \cdot \frac{1}{\widetilde{u}^n(s,t)} \le \varepsilon^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)} d\theta}$$

Multiplying by $e^{-\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)} d\theta}$ we have

$$-\varepsilon \le \left(\widetilde{u} \cdot e^{-\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)} d\theta}\right)'_s - e^{-\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)} d\theta} \cdot \frac{\widetilde{f}(s,t)}{\widetilde{p}(s,t)} \cdot \frac{1}{\widetilde{u}^n(s,t)} \le \varepsilon$$

Integrating with respect to s, we have

$$-\varepsilon(s-a) \le \widetilde{u} \cdot e^{-\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)} d\theta} - \widetilde{\psi}(t) - \int_a^s e^{-\int_a^\theta \frac{\widetilde{r}(\tau,t)}{\widetilde{p}(\tau,t)} d\tau} \cdot \frac{\widetilde{f}(\theta,t)}{\widetilde{p}(\theta,t)} \cdot \frac{1}{\widetilde{u}^n(\theta,t)} d\theta \le \varepsilon(s-a).$$

Multiplying by $e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}$ we have

$$\begin{aligned} -\varepsilon\left(s-a\right)e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta} &\leq \widetilde{u}-e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}\left[\widetilde{\psi}\left(t\right)+\int_a^s e^{-\int_a^\theta \frac{\widetilde{r}(\tau,t)}{\widetilde{p}(\tau,t)}d\tau}\frac{\widetilde{f}}{\widetilde{p}}\frac{1}{\widetilde{u}^n\left(\theta,t\right)}d\theta\right] \\ &\leq \varepsilon\left(s-a\right)e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}. \end{aligned}$$

Hence

$$\left|\widetilde{u}-e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}\left[\widetilde{\psi}\left(t\right)+\int_a^s e^{-\int_a^\theta \frac{\widetilde{r}(\tau,t)}{\widetilde{p}(\tau,t)}d\tau}\cdot\frac{\widetilde{f}\left(\theta,t\right)}{\widetilde{p}\left(\theta,t\right)}\cdot\frac{1}{\widetilde{u}^n\left(\theta,t\right)}d\theta\right]\right|\leq\varepsilon\left(b-a\right)e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}.$$

It can be easily show that

$$\widetilde{w}\left(s,t\right)=e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}\left[\widetilde{\psi}\left(t\right)+\int_a^s e^{-\int_a^\theta \frac{\widetilde{r}(\tau,t)}{\widetilde{p}(\tau,t)}d\tau}\cdot\frac{\widetilde{f}\left(\theta,t\right)}{\widetilde{p}\left(\theta,t\right)}\cdot\frac{1}{\widetilde{w}^n\left(\theta,t\right)}\right]$$

is a solution of the equation (2.7) with initial condition (2.8)

We consider the difference

$$\left|\widetilde{u}\left(s,t\right)-\widetilde{w}\left(s,t\right)\right|\leq\left|\widetilde{u}\left(s,t\right)-e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}\left[\widetilde{\psi}\left(t\right)+\int_a^s e^{-\int_a^\theta \frac{\widetilde{r}(\tau,t)}{\widetilde{p}(\tau,)}d\tau}\cdot\frac{\widetilde{f}\left(\theta,t\right)}{\widetilde{p}\left(\theta,t\right)}\cdot\frac{1}{\widetilde{u}^n\left(\theta,t\right)}d\theta\right]\right|$$

$$\left|e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}\cdot\int_a^s e^{-\int_a^\theta \frac{\widetilde{r}(\tau,t)}{\widetilde{p}(\tau,)}d\tau}\cdot\left(\frac{\widetilde{f}\left(\theta,t\right)}{\widetilde{p}\left(\theta,t\right)}\cdot\frac{1}{\widetilde{u}^n\left(\theta,t\right)}-\frac{\widetilde{f}\left(\theta,t\right)}{\widetilde{p}\left(\theta,t\right)}\cdot\frac{1}{\widetilde{w}^n\left(\theta,t\right)}\right)d\theta\right|$$

$$\leq\varepsilon\left(b-a\right)e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}+e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}\int_a^s e^{-\int_a^\theta \frac{\widetilde{r}(\tau,t)}{\widetilde{p}(\tau,t)}d\tau}\frac{\widetilde{f}}{\widetilde{p}}\left|\frac{1}{\widetilde{u}^n\left(\theta,t\right)}-\frac{1}{\widetilde{w}^n\left(\theta,t\right)}\right|d\theta$$

Using (1.3) we obtain

$$\begin{aligned} \left|\widetilde{u}\left(s,t\right)-\widetilde{w}\left(s,t\right)\right| &\leq\varepsilon\left(b-a\right)e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}+Le^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}\int_a^s e^{-\int_a^\theta \frac{\widetilde{r}(\tau,t)}{\widetilde{p}(\tau,t)}d\tau}\left|\widetilde{u}\left(\theta,t\right)-\widetilde{w}\left(\theta,t\right)\right|d\theta \\ &=\varepsilon\left(b-a\right)e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}+L\int_a^s e^{\int_\theta^s \frac{\widetilde{r}(\tau,t)}{\widetilde{p}(\tau,t)}d\tau}\left|\widetilde{u}\left(\theta,t\right)-\widetilde{w}\left(\theta,t\right)\right|d\theta \end{aligned}$$

Using Gronwall's inequality we obtain

$$\begin{aligned} \left|\widetilde{u}\left(s,t\right)-\widetilde{w}\left(s,t\right)\right| &\leq\varepsilon\left(b-a\right)e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}\cdot e^{L\int_a^s e^{\int_\theta^s \frac{\widetilde{r}(\tau,t)}{\widetilde{p}(\tau,t)}d\tau}d\theta}\leq\varepsilon\left(b-a\right)e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}\cdot e^{L\int_a^s e^{(s-\theta)M}d\theta} \\ &=\varepsilon\left(b-a\right)e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}\cdot e^{-\frac{L}{M}\left(1-e^{(s-a)M}\right)}=\varepsilon\left(b-a\right)e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}\cdot e^{-\frac{L}{M}}. \end{aligned}$$

Consequently

$$\begin{aligned} \left|u\left(x,y\right)-w\left(x,y\right)\right| &= \left|\widetilde{u}\left(s,t\right)-\widetilde{w}\left(s,t\right)\right|\leq\varepsilon\left(b-a\right)e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)}d\theta}\cdot e^{-\frac{L}{M}}= \\ &= \varepsilon\left(b-a\right)e^{\int_a^x \frac{r(\theta,\varphi(\theta)+y-\varphi(x))}{p(\theta,\varphi(\theta)+y-\varphi(x))}d\theta}\cdot e^{-\frac{L}{M}}. \end{aligned}$$

We denote $c_{f,\phi}=\left(b-a\right)e^{-\frac{L}{M}}$ Hence

$$\left|u\left(x,y\right)-w\left(x,y\right)\right|\leq c_{f,\phi}\varepsilon\phi\left(x,y\right),\forall\left(x,y\right)\in D.$$

that is the equation (1.1), with initial condition (1.2), is Ulam-Hyers-Rassias stable with respect to ϕ. □

Remark 2.1. We suppose now $b = \infty$. We have

$$|\widetilde{u}(s,t) - \widetilde{w}(s,t)| \leq \varepsilon (s-a)\, e^{\int_a^s \frac{\widetilde{r}(\theta,t)}{\widetilde{p}(\theta,t)} d\theta} \cdot e^{-\frac{L}{M}} \leq \varepsilon (s-a)\, e^{M(s-a)} \cdot e^{-\frac{L}{M}}. \tag{2.9}$$

Setting $s \to \infty$ in (2.9), we have $\lim\limits_{s\to\infty} (s-a)\, e^{M(s-a)} = 0$, so

$$\lim_{s\to\infty} |\widetilde{u}(s,t) - \widetilde{w}(s,t)| = 0.$$

Consequently the problem is asymptotic stable.

Remark 2.2. If $r(x,y) = p(x,y) \cdot r_1(x)$ and $n = 0$, the quasilinear differential equation (1.1) becomes the partial differential equation

$$p(x,y)\frac{\partial u}{\partial x} + q(x,y)\frac{\partial u}{\partial y} = p(x,y)\, r_1(x)\, u(x,y) + f(x,y).$$

Hyers-Ulam stability of this equation was studied in [16].

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